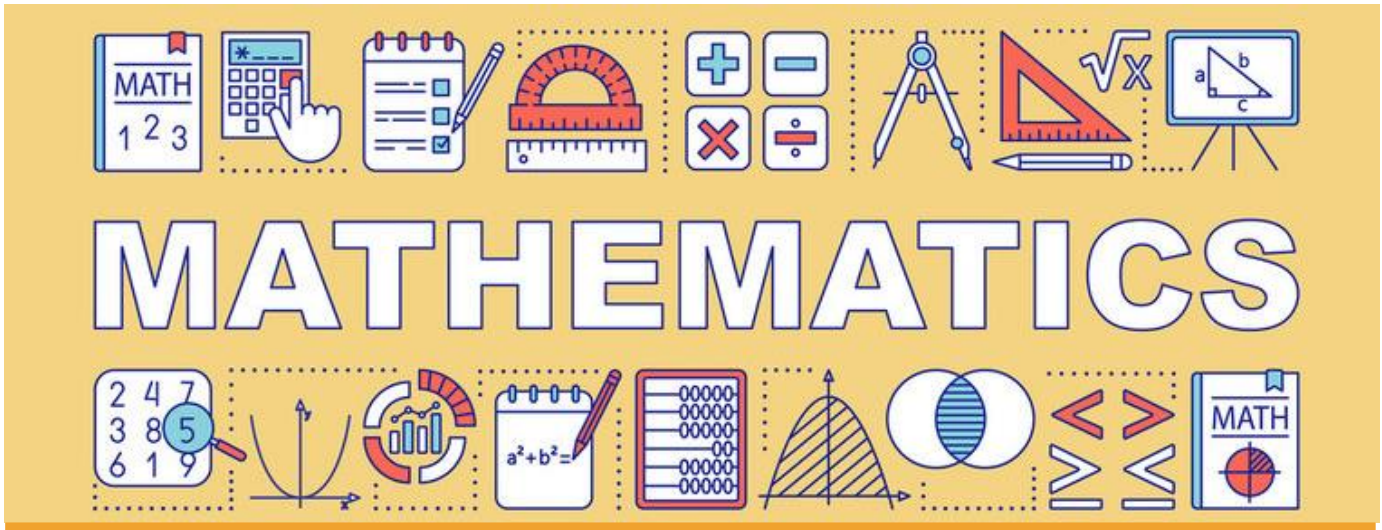
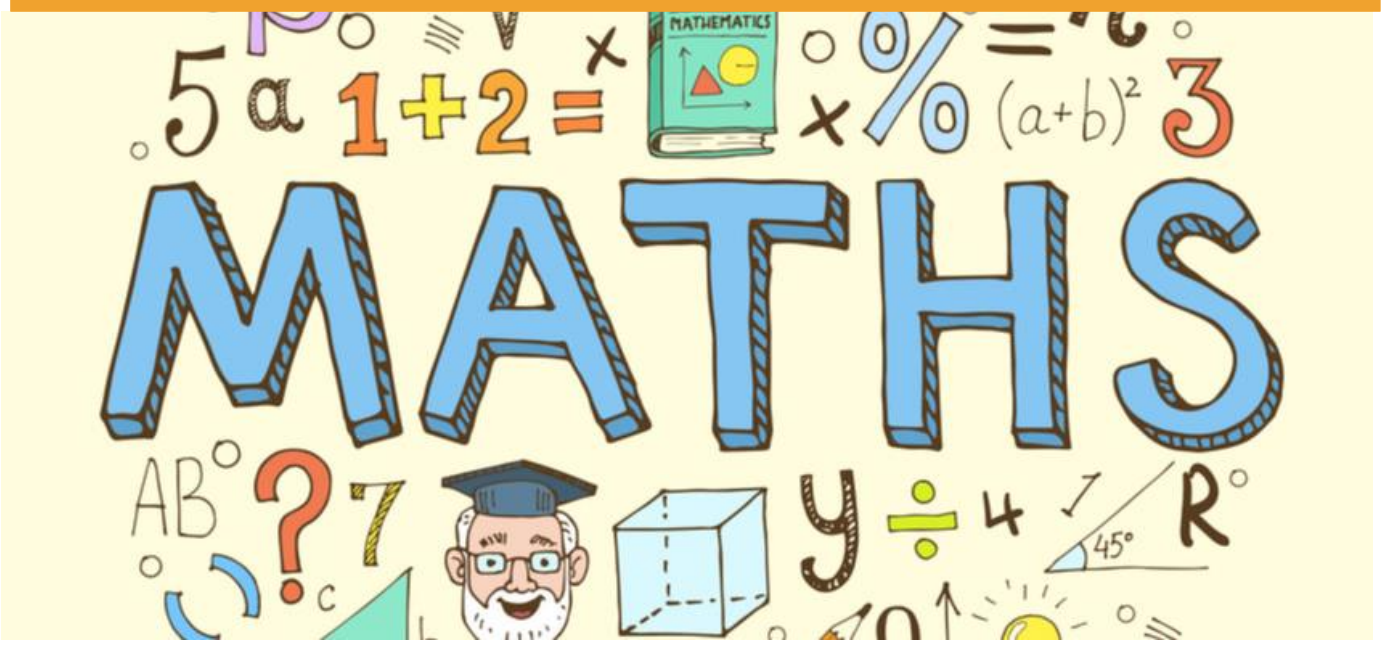


ENGLISH FOR MATHEMATICS STUDENTS

Навчальний посібник.

Укладачі:

Куліш І.М., Зінченко А.В., Некоз І.В.



МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ
ЧЕРКАСЬКИЙ НАЦІОНАЛЬНИЙ УНІВЕРСИТЕТ
ІМЕНІ БОГДАНА ХМЕЛЬНИЦЬКОГО

ENGLISH FOR MATHEMATICS STUDENTS

Черкаси – 2025

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English for Mathematics Students: Навчальний посібник для студентів спеціальності «Математика». Укладачі: Куліш І.М., Зінченко А.В., Некоз І.В.– Друк ФОП Нечитайло О.Ф. Україна, м. Черкаси. - 2025. – 63 с.

Навчальний посібник сформований у відповідності до програми з іноземної мови за професійним спрямуванням для студентів 1 курсу спеціальності «Математика» та інших математичних спеціалізацій.

Навчальний матеріал розташовано за тематичним принципом (за розділами) та має комплексний характер. Два розділи охоплюють навчальний матеріал з іноземної мови професійного спрямування та зосереджуються на застосуванні отриманих знань на практиці.

Робота за темою в межах одного розділу організовується на базі тексту професійного, лексичних вправ та завдань комунікативного характеру професійного та загального спрямування, текстів для додаткового читання, системи вправ репродуктивного та творчого характеру. Посібник вміщує завдання для самостійної роботи студентів та вправи для контролю умінь та навичок студентів.

Затверджено до друку на засіданні вченої ради
Черкаського Національного університету
імені Богдана Хмельницького
протокол № 1 від 19.02.2025

ПЕРЕДМОВА

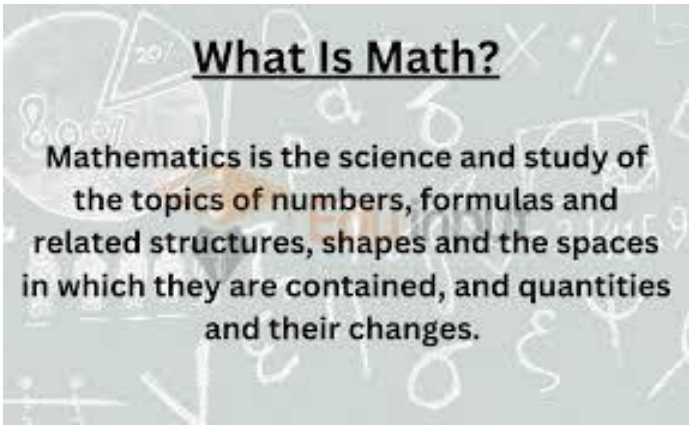
Навчальний посібник призначається для студентів спеціальності «Математика» та інших математичних спеціалізацій. Мета посібника – формувати у студентів навички читання та розуміння літератури фахової тематики на основі активізації знань професійної лексики та основ граматики. Поряд з цим у посібнику реалізується мета комплексного оволодіння студентами всіма видами мовленнєвої діяльності на основі комунікативного підходу до вивчення іноземної мови. Посібник базується на різноманітному автентичному матеріалі, який підібрано з урахуванням комунікативного підходу у вивченні англійської мови та спрямовано на розвиток позитивної мотивації до вивчення іноземної мови.

Навчальний матеріал професійного спрямування посібника розподілено на два тематичних розділи “Mathematics” та “Number Theory”, текстовий матеріал яких згруповано за такими темами професійного інтересу студентів, як вступ до математики, алгебра, геометрія, аналіз та математичні теорії, а також теорія чисел, її історичний аспект та значення, тощо. Кожний розділ, в свою чергу, вміщує основний текст професійного спрямування та комплекс вправ, які з різних сторін висвітлюють тему, та текстів, які несуть додаткову інформацію за певною тематикою. Кожен розділ закінчується тематичним термінологічних словником.

Комунікативні вправи мають різноплановий вплив на навчальний процес та стимулюють пізнавальний інтерес особистості студента.

Unit I
Mathematics
Lesson 1
Introduction to Mathematics

(From Thinking of a Career in Applied Mathematics? | SIAM". www.siam.org. Retrieved 2018-07-30.
Wigner, Eugene (February 1960). "The Unreasonable Effectiveness of Mathematics in the Natural Sciences". Communications in Pure and Applied Mathematics. 13 (1): 1–14. Bibcode:1960CPAM...13....1W. doi:10.1002/cpa.3160130102. ISSN 0010-3640. S2CID 6112252. Archived from the original on 2018-08-10. Retrieved 2018-08-07.)



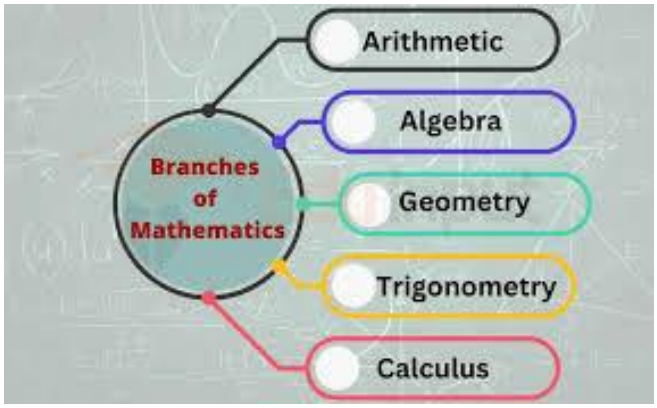
Mathematics is the study of numbers, shapes, and patterns. The word comes from the Greek máthema, meaning "science, knowledge, or learning", and is sometimes shortened to math or maths.

It is the study of:

- Numbers: including how things can be counted.
- Structure: including how things are organized, but also how they can be or could have been. This subfield is usually called algebra.
- Place: where things are, and spatial arrangement, including arrangements of spaces themselves. This subfield is usually called geometry.
- Change: how things become different. This subfield is usually called analysis.

Applied math is useful for solving problems in the real world. People working in business, science, engineering, and construction use mathematics.

1. Find the English equivalents in the text:



Вивчення чисел, походити з, бути скороченим до, рахувати, підрозділ (підгалузь), просторове розташування, називати, прикладна математика, наука, інженерія, будівництво, використовувати.

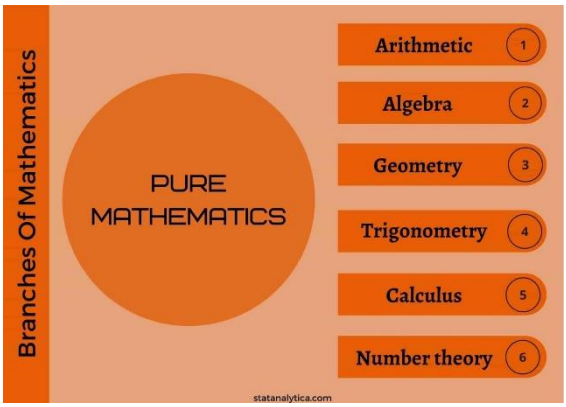
2. Translate the following terms and phrases:

Mathematics, number, shape, pattern, structure, subfield, algebra, spatial arrangement, geometry, analysis, applied math, business, science, engineering, construction.

3. Match the terms and their definitions:

1	algebra	involves the application of mathematics to problems which arise in various areas, e.g., science, engineering or other diverse areas	
2	geometry	the science and study of quality, structure, space, and change	
3	analysis	the branch of mathematics dealing with limits and related theories, such as differentiation, integration, measure, infinite series, and analytic functions	
4	mathematics	a branch of mathematics concerned with properties of space such as the distance, shape, size, and relative position of figures	
5	applied mathematics	a branch of mathematics that uses mathematical statements to describe relationships between things that vary	

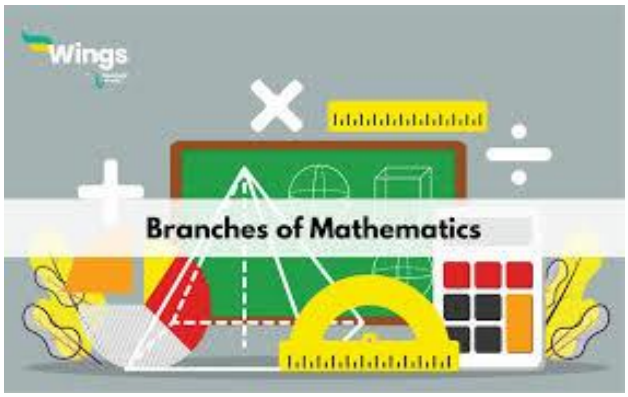
4. Complete the sentences using the following terms and phrases:



mathematics
algebra
mathematical analysis
geometrical concepts
applied mathematics

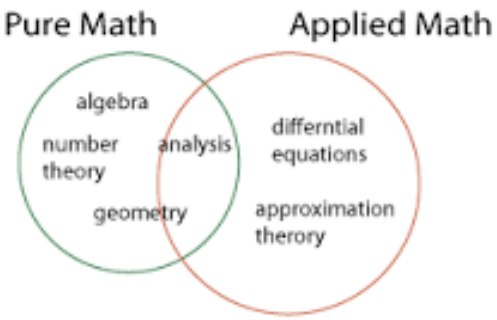
1) Mathematics focuses on abstract concepts and theoretical problems, while _____emphasizes solving real-world problems using mathematical techniques.
2) _____is the science of structure, order, and relation that has evolved from elemental practices of counting, measuring, and describing the shapes of objects.
3) _____serves as a common foundation for many research areas of pure and applied mathematics.
4) _____involves variables like x, y, z, and mathematical operations like addition, subtraction, multiplication, and division to form a meaningful mathematical expression.
5) The basic _____are dependent on three basic concepts: the point, line and plane.

5. Agree or disagree with the statements:



1) The word mathematics comes from the Roman word meaning "science".
2) Mathematics is the study of numbers, shapes, and patterns.
3) Algebra is the subfield of mathematics about spatial arrangement.
4) Geometry is about structure: including how things are organized.
5) The subfield about change and how things become different is usually called analysis.
6) Numbers are useful for solving problems in the real world.

6. Read the text and put 2 questions to it for the group discussion:

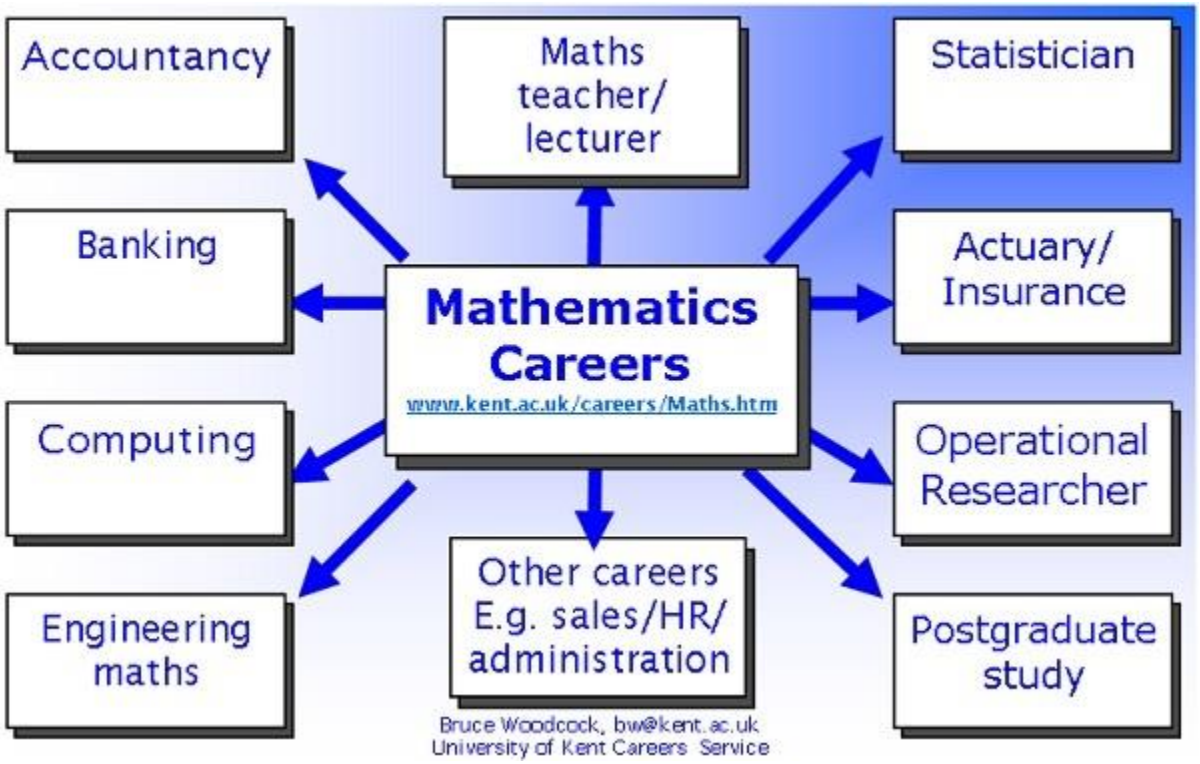


In the world of math, there are different paths students can take, whether that is choosing pure mathematics or applied mathematics. What is the difference between pure math vs. applied math?

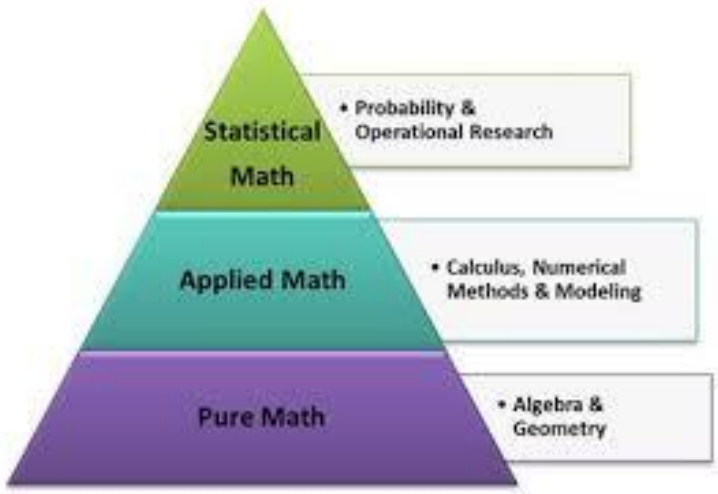
"Pure" math describes studying or working in the field of theoretical or abstract mathematics to further mathematical knowledge.

Applied mathematics is the application of mathematical methods in various fields, such as physics, computer science, engineering, business, biology, information technology, and much more.

7. **Look at the scheme and say what careers students majoring in mathematics can choose and what you would choose:**



8. **Answer the questions for summary:**

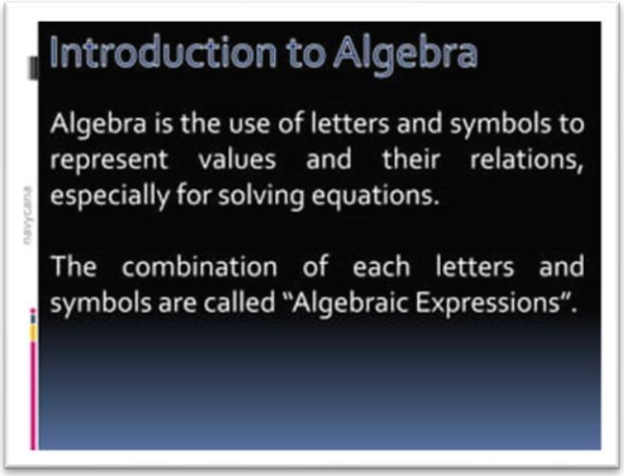


1) What does mathematics study?
2) What is the origin of the word mathematics?
3) What studies does mathematics include?
4) What does algebra study?
5) What does geometry focus on?

6) What subfield of mathematics studies change of things and how things become different?
7) What is applied mathematics useful for?
8) What is the difference between pure math and applied math?
9) What careers can students majoring in mathematics choose?
10) What job relating to mathematics would you choose?

Lesson 2
Algebra

(From "Algebra Introduction". Math is Fun. Retrieved from: https://www.mathsisfun.com/algebra/introduction.html 11 April 2013.)



Algebra means "reunion of broken parts" from the Arabic language. Algebra is a part of mathematics. It uses variables to represent a value that is not yet known or can be replaced with any value. When an equal sign (=) is used, this is called an equation. A very simple equation using a variable is: 2+3=x . In this example, x=5, or it could also be said that "x equals five". This is called solving for x.

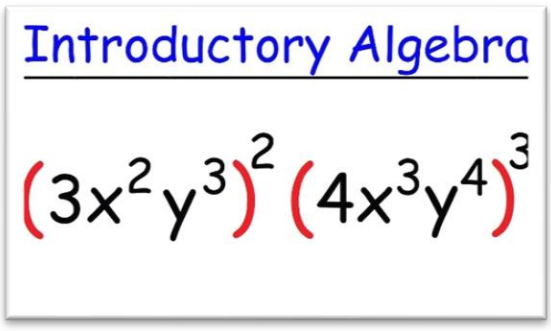
Besides equations, there are inequalities (*less than* and *greater than*). A special type of equation is called the function. This is often used in making graphs because it always turns one input into one output.

Algebra can be used to solve real problems because the rules of algebra work in real life and numbers can be used to represent the values of real things. Physics, engineering and computer programming are areas that use algebra all the time. It is also useful to know in surveying, construction and business, especially accounting.

People who do algebra use the rules of numbers and mathematical operations used on numbers. The simplest are adding, subtracting, multiplying, and dividing. More advanced operations involve exponents, starting with squares and square roots.

Algebra was first used to solve equations and inequalities. Two examples are linear equations (the equation of a straight line) and quadratic equations, which has variables that are squared (multiplied by itself).

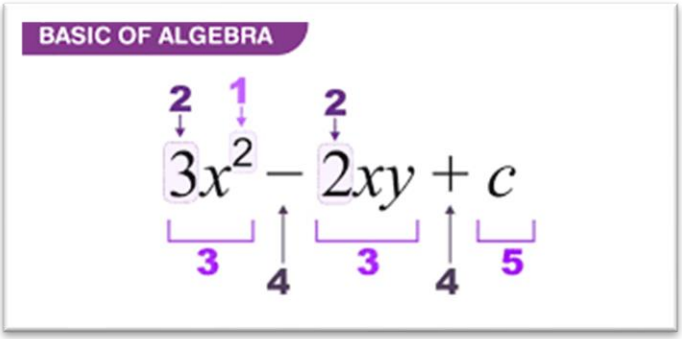
1. Find the English equivalents to the text:



Означати, використовувати змінні, представляти, значення, заміняти, знак рівності, розв'язування для х, крім рівнянь, створення графіків, перетворювати, вирішувати реальні проблеми, фізика, інженерія, комп'ютерне програмування, геодезії, будівництві,

бізнесі, бухгалтерія, займатися алгеброю, додавання, віднімання, множення, ділення, складні операції, показники степеня, квадрат, квадратний корінь, лінійне рівняння, квадратне рівняння, множити.

2. Translate the following terms and phrases:

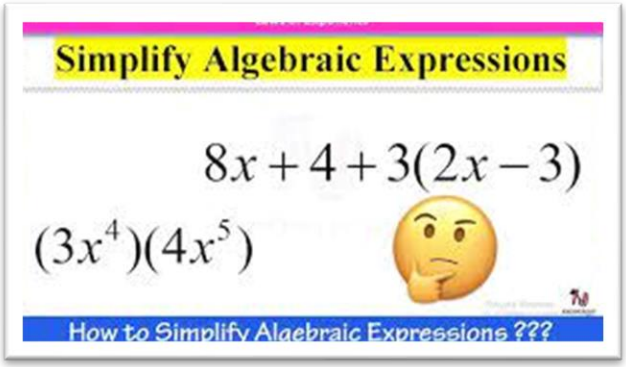


Variable, to represent a value, equation, inequality, graph, physics, engineering, computer programming, surveying, construction, accounting, adding, subtracting, multiplying, dividing, exponent, square, square root, linear equation, quadratic equation.

3. Match the notions and their definitions:

1	physics	the study of using scientific principles to design and build machines, structures, and other things, including bridges, roads, vehicles, and buildings	
2	engineering	the technique, profession, art, and science of determining the terrestrial two-dimensional or three-dimensional positions of points and the distances and angles between them	
3	surveying	the process of recording financial transactions pertaining to a business	
4	construction	the branch of science that deals with the structure of matter and how the fundamental constituents of the universe interact	
5	accounting	the art and science of forming objects, systems, or organizations	

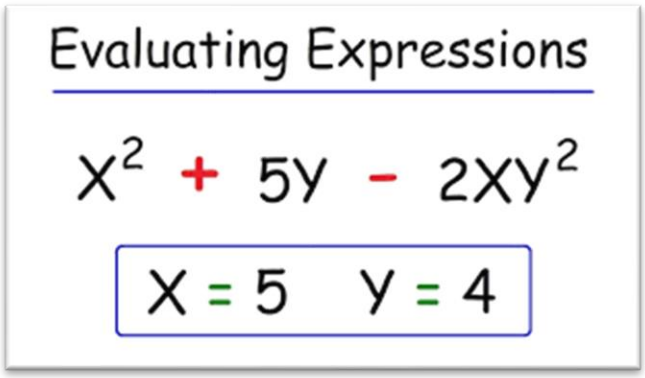
4. Complete the sentences with the following words and phrases:



construction
surveying
physics
accounting
engineering

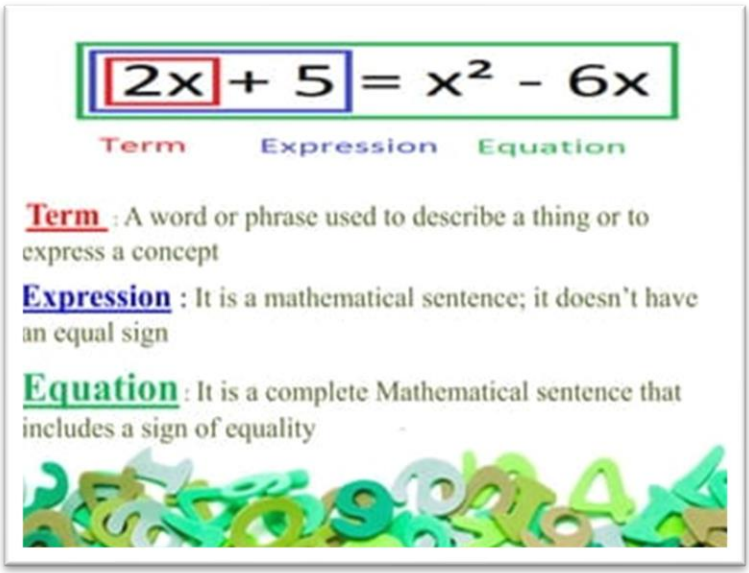
1) _____is the science or profession of developing and using nature's power and resources in ways that are useful to people.
2) _____is the process where contractors build structures that serve a particular purpose.
3) The main objective of _____is to determine the relative position of any objects or points on the earth.
4) The main goal of _____is to record and report a company's financial transactions, financial performance, and cash flows.
5) _____itself is studying the natural world and the interactions between objects and energy in any given environment.

5. Agree or disagree with the statements:



1) The word algebra from the English language means “reunion of broken parts”.
2) Algebra is a part of arithmetic.
3) When an equal sign (=) is used, this is called an equation.
4) A special type of equation is called the function.
5) Equation is often used in making graphs.
6) Algebra cannot be used to solve real problems.
7) The simplest operations ai algebra are adding, subtracting, multiplying, and dividing.
8) Quadratic equations are the equation of a straight line.

6. Read and translate the definitions:



7. Read the expression and answer the questions:

Language of Algebra

2. Given the expression $3a+7b+1$

a) How many terms are there?

b) What are the variable terms?

c) What is the constant term?

8. Look at the side and answer the question:

What should we do to translate a verbal phrase into an algebraic expression?

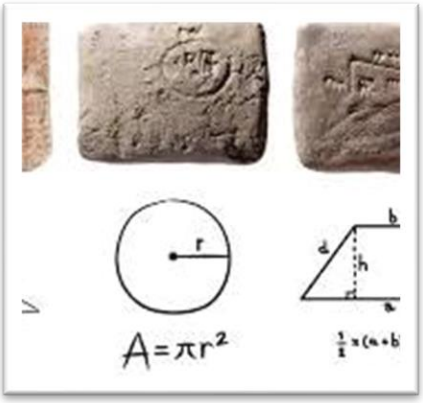
Lesson 1: **Algebraic Expressions**

Write Expressions

To translate a verbal phrase into an algebraic expression, the first step is to define a variable. When you define a variable, you choose a variable to represent an unknown quantity.

9. Read the text and put 2 questions to discuss them in the group:

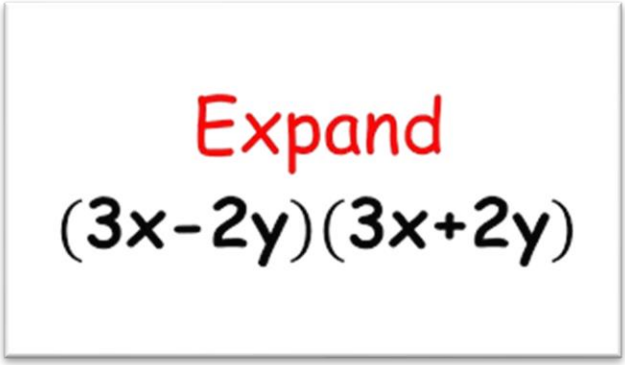
(From "History of Algebra". UCS Louisiana. Archived from the original on 9 October 2014. Retrieved from: https://web.archive.org/web/20141009100628/http://www.ucs.louisiana.edu/~sxw8045/history.htm 11 April 2013)



Early forms of algebra were developed by the Babylonians and Greek geometers such as Hero of Alexandria. However, the word "algebra" is a Latin form of the Arabic word Al-Jabr ("casting") and comes from a mathematics book Al-Maqala fi Hisab-al Jabr wa-al-Muqabilah, ("Essay on the Computation of Casting and Equation") written in the 9th century by a Persian mathematician, Muhammad ibn Mūsā al-

Khwārizmī, who was a Muslim born in Khwarizm in Uzbekistan. He flourished under Al-Ma'moun in Baghdad, Iraq through 813-833 CE, and died around 840 CE. The book was brought into Europe and translated into Latin in the 12th century. The book was then given the name “Algebra”. (The ending of the mathematician's name, al-Khwarizmi, was changed into a word easier to say in Latin, and became the English word algorithm).

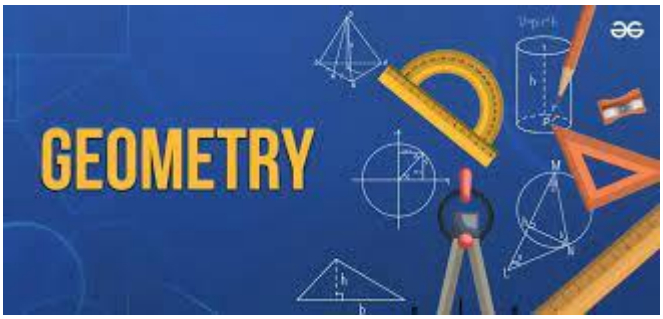
10.Answer the following questions for summary:



1) What does the word algebra mean?
2) What does algebra use variables for?
3) What is an equation?
4) What is function?
5) What is often used in making graphs?
6) Why can algebra be used to solve real problems?
7) What areas use algebra?
8) What rules do people who do algebra use?
9) What are the simplest operations in algebra?
10) What are more advanced operations in algebra?

Lesson 3
Geometry

(From encyclopedia Brittanica. Retrieved from: https://www.britannica.com/science/geometry)



Geometry is the branch of mathematics concerned with the shape of individual objects, spatial relationships among various objects, and the properties of surrounding space. It is one of the oldest branches of mathematics. Its name is derived from Greek words meaning “Earth measurement”.

Eventually it was realized that geometry need not be limited to the study of flat surfaces (plane geometry) and rigid three-dimensional objects (solid geometry) but that even the most abstract thoughts and images might be represented and developed in geometric terms.

In several ancient cultures there developed a form of geometry suited to the relationships between lengths, areas, and volumes of physical objects. This geometry was codified in Euclid’s Elements about 300 BC on the basis of 10 axioms, or postulates, from which several hundred theorems were proved by deductive logic. The Elements epitomized the axiomatic-deductive method for many centuries.

1. Find the English equivalents in the text:

Форма окремих об’єктів, просторові співвідношення, властивості навколишнього простору, вимірювання, плоска поверхня, тверді тривимірні об’єкти, відповідати зв’язкам, довжина, площа, об’єм фізичних об’єктів, бути доведеним, втілювати аксіоматично-дедуктивний метод.

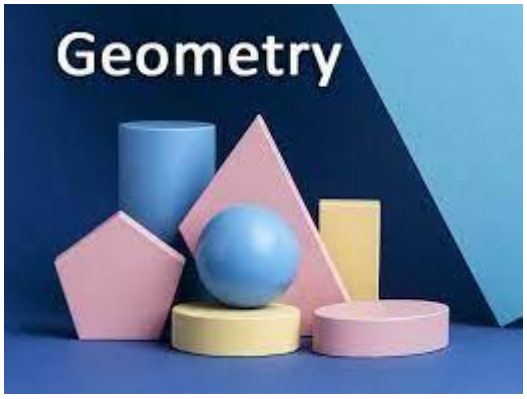
2. Translate the following terms and phrases:

Branch of mathematics, shape, spatial relationships, object, property, surrounding space, measurement, plane geometry, solid geometry, abstract thoughts, geometric term, length, area, volume, physical object, axiom, postulate, theorem, deductive logic, axiomatic-deductive method.

3. Match the terms and their definitions:

1	solid geometry	an unprovable rule or first principle accepted as true because it is self-evident or particularly useful	
2	three-dimensional figure	a shape with two dimensions, such as width and height	
3	plane geometry	a system of measuring flat, two-dimensional shapes	
4	two-dimensional shape	a solid figure or an object or shape that has three dimensions— length, width, and height	
5	axiom	the geometry of three-dimensional Euclidean space	

4. Complete the sentences with the following terms and phrases:



three-dimensional figures
plane geometry
axiom
two-dimensional shape
solid geometry

1) _____includes the measurements of volumes of various solid figures (three-dimensional figures).
2) _____include pyramids, cylinders, cones, spheres, and prisms.
3) An example of a _____is a rectangle or a circle.

4) If you use a protractor to measure the angle of a triangle, you're doing_____.
5) The word _____comes from a Greek word meaning “worthy.”

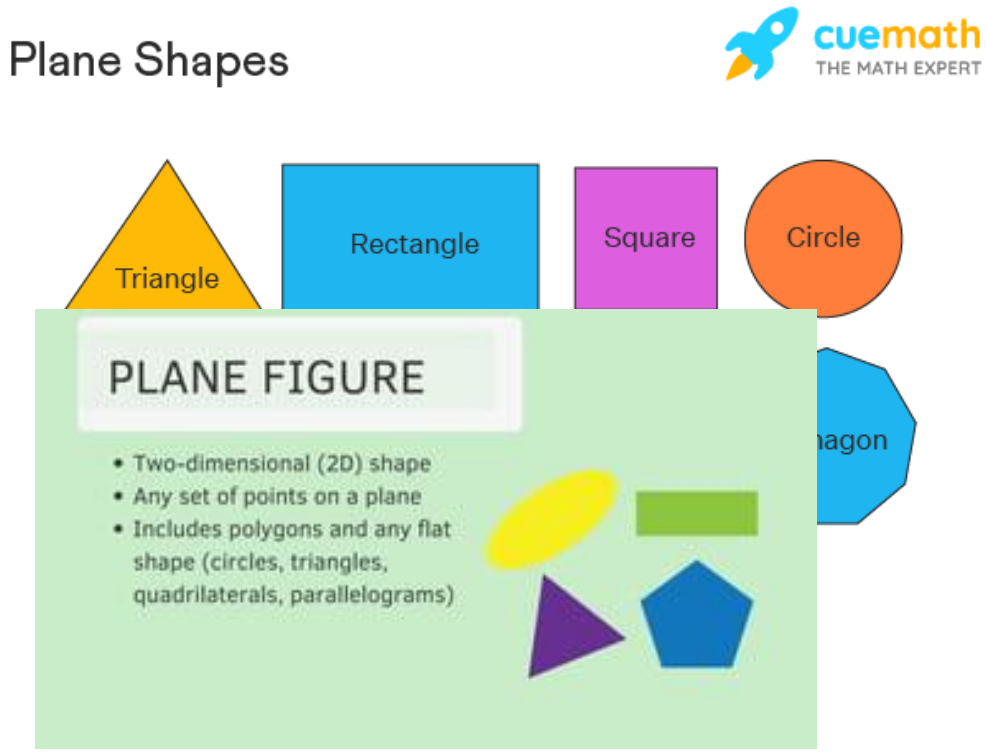
5. Read a paragraph and say what analytic geometry is and who initiated analytic geometry.

Analytic geometry was initiated by the French mathematician René Descartes (1596–1650), who introduced rectangular coordinates to locate points and to enable lines and curves to be represented with algebraic equations. Algebraic geometry is a modern extension of the subject to multidimensional and non-Euclidean spaces.

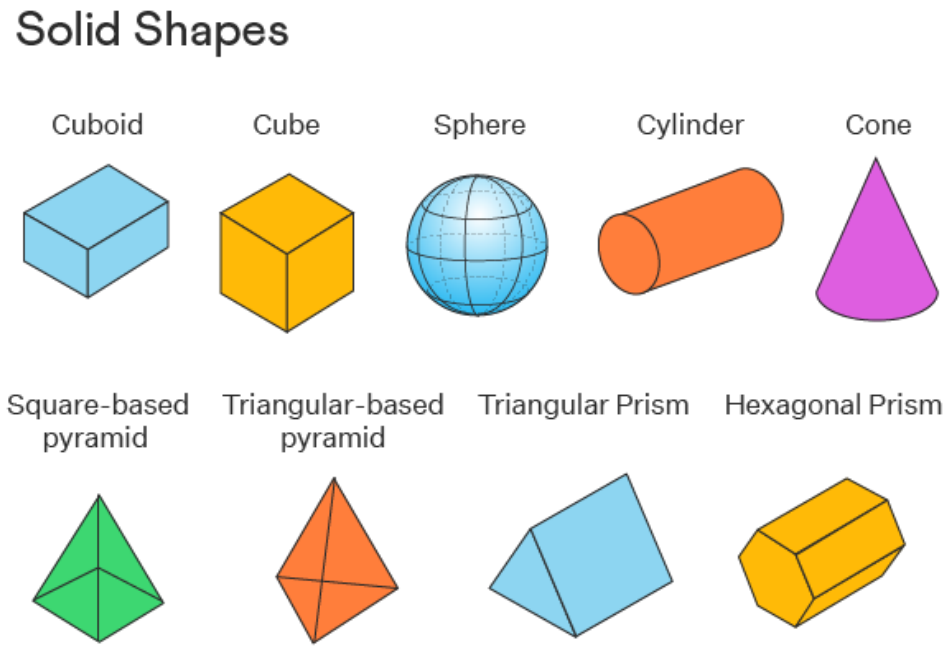
6. Read a paragraph and say what practical problems initiated differential geometry and who characterized it.

The German mathematician Carl Friedrich Gauss (1777–1855), in connection with practical problems of surveying and geodesy, initiated the field of differential geometry. Using differential calculus, he characterized the intrinsic properties of curves and surfaces. For instance, he showed that the intrinsic curvature of a cylinder is the same as that of a plane, as can be seen by cutting a cylinder along its axis and flattening, but not the same as that of a sphere, which cannot be flattened without distortion.

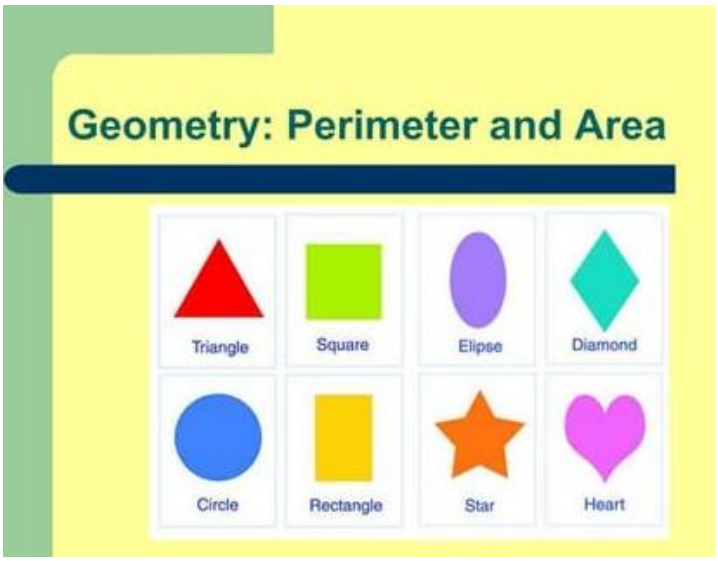
7. Look at the slide and name plane shapes in English and Ukrainian:



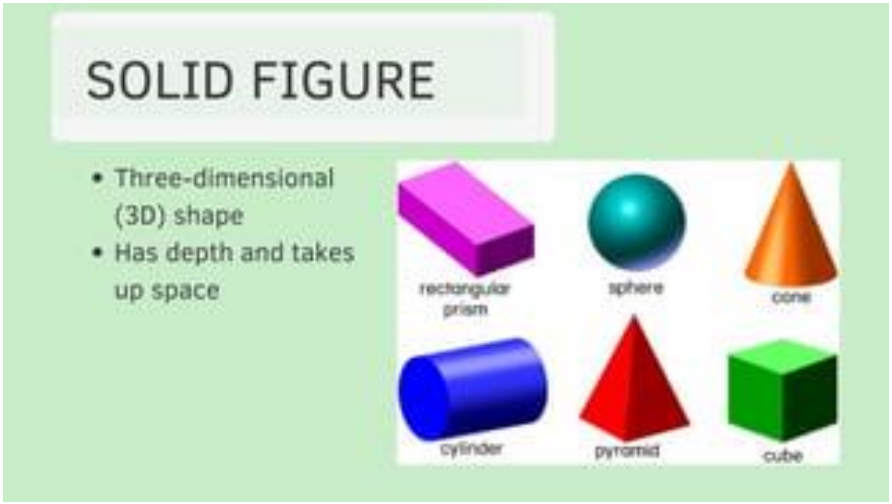
8. Look at the slide and name solid shapes in English and Ukrainian:



9. Agree or disagree with the statements:

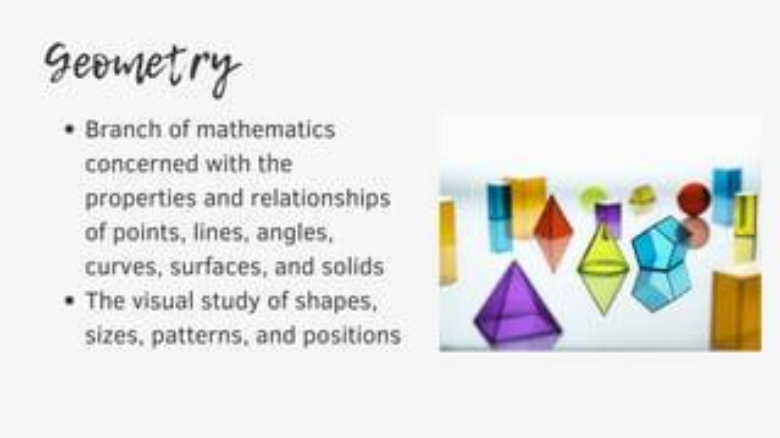


1) Geometry is the branch of algebra concerned with the shape of individual objects.
2) Geometry is a new branch of mathematics.
3) The word geometry is derived from Latin word meaning "Earth measurement".
4) Analytic geometry was initiated by the German mathematician Carl Friedrich Gauss.
5) Differential geometry was initiated by the French mathematician René Descartes.



6) Using differential calculus, Carl Friedrich Gauss characterized the intrinsic properties of curves and surfaces.
7) Differential geometry was initiated in connection with practical problems of surveying and geodesy.

10.Answer the following questions for summary:

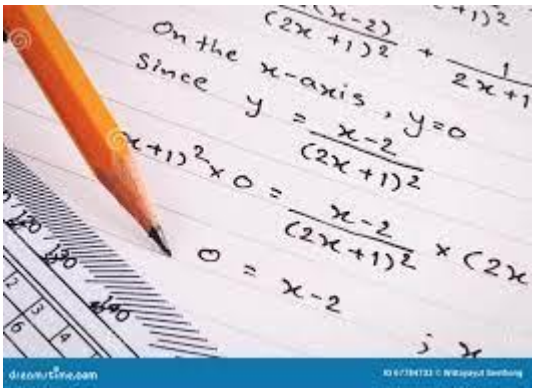


1) What branch of mathematics is geometry?
2) Is geometry new branch of mathematics?
3) What does the word geometry mean?
4) What does plane geometry study?
5) What does solid geometry study?
6) What is two-dimensional shape?
7) What are the examples of plane shapes?
8) What is three-dimensional shape?
9) What are the examples of solid shapes?

Lesson 4

Analysis

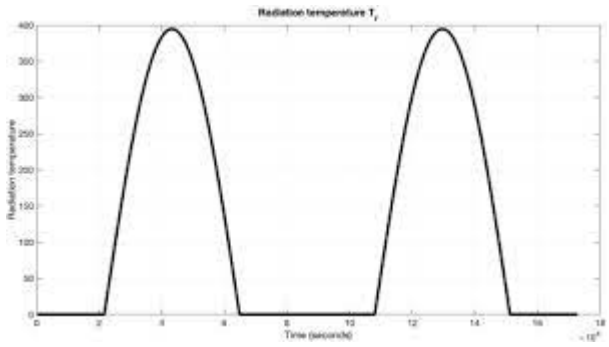
(From Encyclopedia Brittanica. Retrieved from: https://www.britannica.com/summary/analysis-mathematics)



Analysis is a field of mathematics that incorporates the methods of algebra and calculus—specifically of limits, continuity, and infinite series—to analyze classes of functions and equations having general properties (e.g., differentiability). Analysis builds on the work of G.W. Leibniz and Isaac Newton by exploring the applications of the derivative and the integral. Several distinct but related subfields have developed, including the calculus of variations, differential equations, Fourier analysis, complex analysis, vector and tensor analysis, real analysis, and functional analysis.

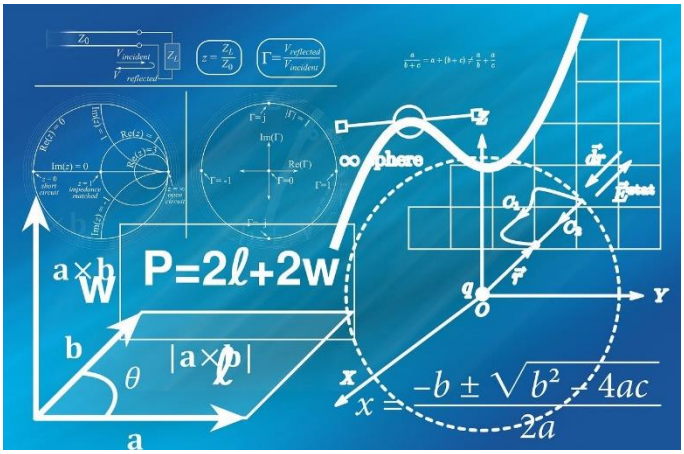
Numerical analysis is a branch of applied mathematics that studies methods for solving complicated equations using arithmetic operations, often so complex that they require a computer, to approximate the processes of analysis (i.e., calculus). The arithmetic model for such an approximation is called an algorithm, the set of procedures the computer executes is called a program, and the commands that carry out the procedures are called code. An example is an algorithm for deriving π by calculating the perimeter of a regular polygon as its number of sides becomes very large. Numerical analysis is concerned not just with the numerical result of such a process but with determining whether the error at any stage is within acceptable bounds.

1. Find the English equivalents in the text:



Галузь математики, поєднувати методи, обмеження, неперервність, нескінченний ряд, загальна властивість, диференційованість, дослідити застосування похідної, окремі галузі, пов’язані галузі, варіаційне числення, тензорний аналіз, реальний аналіз, числовий аналіз, розділ прикладної математики, периметр правильного багатокутника, визначити помилку, в прийнятих межах.

2. Translate the following terms and phrases:

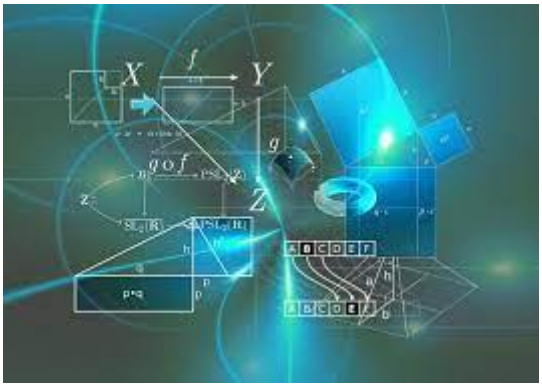


Algebra, calculus, limits, continuity, infinite series, differentiability, derivative, the integral, the calculus of variations, differential equations, Fourier analysis, complex analysis, vector analysis, tensor analysis, real analysis, functional analysis, numerical analysis, applied mathematics, complicated equations, arithmetic operations, often so complex that they require a computer, to approximate the processes of analysis, algorithm, program, code, the perimeter of a regular polygon, the error.

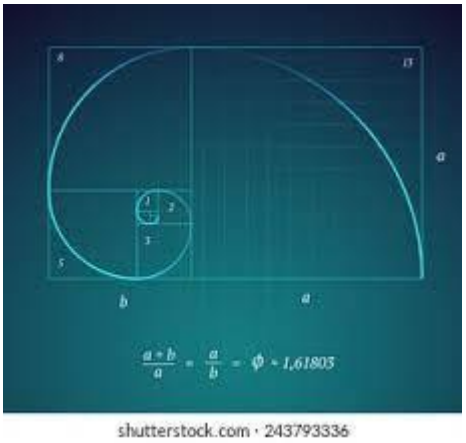
3. Match the notions and their definition:

1	applied mathematics	a procedure for solving a mathematical problem (as of finding the greatest common divisor) in a finite number of steps that frequently involves repetition of an operation	
2	numerical analysis	the application of mathematics to problems which arise in various areas	
3	algorithm	a sequence or set of instructions in a programming language for a computer to execute	
4	program	the set of instructions, or a system of rules, written in a particular programming language	
5	code	an area of mathematics and computer science that creates, analyzes, and implements algorithms for obtaining numerical solutions to problems involving continuous variables	

4. Complete the sentences using the following terms and phrases:



program
applied mathematics
coding
algorithm
numerical analysis



1) _____is good for two things: (1) identifying and understanding numerical errors; (2) learning numerical computation algorithms for solving differential equations, linear systems, differentiation, and integration.

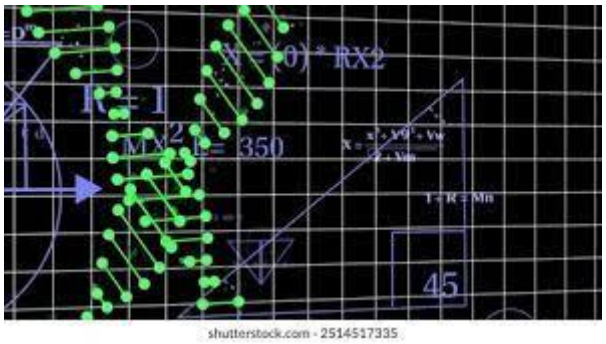
2) An_____, especially in mathematics, is a step-by-step procedure that can be used to solve computations or other mathematical problems.

3) A computer_____, also known as an application, is a system of one or more coded commands that you use to complete an action on your device.

4) _____is used in various fields, such as physics, computer science, engineering, business, biology, information technology, etc.

5) creates a set of instructions for computers to follow.

5. Agree or disagree with the statements:



1) Algebra is a field of mathematics that incorporates the methods of algebra and calculus.

2) Analysis builds on the work of G.W. Leibniz and Isaac Newton.

3) Mathematics is a branch of applied mathematics that studies methods for solving complicated equations.

4) The arithmetic model for such an approximation is called a program.

5) The set of procedures the computer executes is called an algorithm.

6) The commands that carry out the procedures are called code.

6. Read the text and answer the following questions:

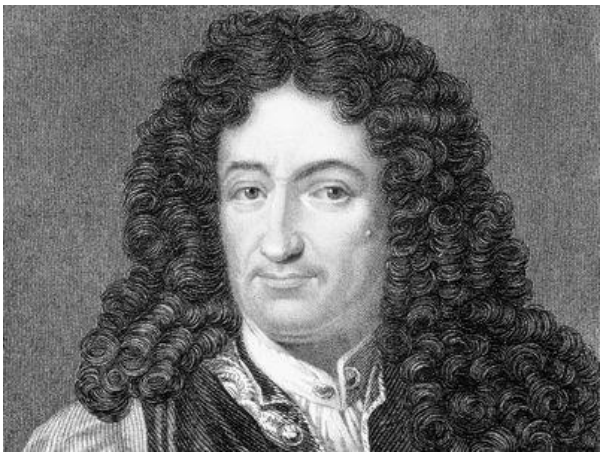
1) Who developed calculus?

2) What is the importance of calculus nowadays?

3) What do you know about Gottfried Wilhelm Leibniz?

Two Mathematicians

(From encyclopedia Britannica. Retrieved from: https://www.britannica.com/science/calculus-mathematics)



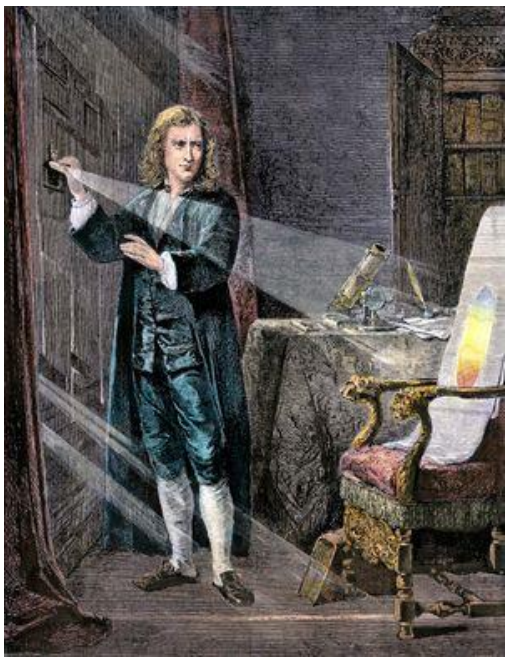
Two mathematicians, Isaac Newton of England and Gottfried Wilhelm Leibniz of Germany, share credit for having independently developed the calculus in the 17th century. Calculus is now the basic entry point for anyone wishing to study physics, chemistry, biology, economics, finance, or actuarial science. Calculus makes it possible to solve problems as diverse as tracking the position of a space shuttle or predicting the pressure building up behind a dam as the water rises. Computers have become a valuable tool for solving calculus problems that were once considered impossibly difficult.

Gottfried Wilhelm Leibniz (born June 21, 1646, Leipzig, Germany —died November 14, 1716, Hanover, Germany) was a German philosopher, mathematician, and political adviser, important both as a metaphysician and as a logician and distinguished also for his independent invention of the differential and integral calculus.

7. Read the text and put 4 questions for the group discussion:

Isaac Newton

(From encyclopedia Britannica. Retrieved from: https://www.britannica.com/biography/Isaac-Newton)



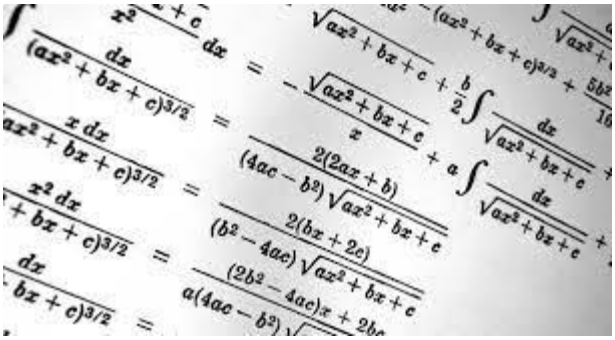
Isaac Newton (born December 25, 1642, Woolsthorpe, Lincolnshire, England—died March 20, 1727, London) was an English physicist and mathematician who was the culminating figure of the Scientific Revolution of the 17th century.

In optics, his discovery of the composition of white light integrated the phenomena of colours into the science of light and laid the foundation for modern physical optics.

In mechanics, his three laws of motion, the basic principles of modern physics, resulted in the formulation of the law of universal gravitation.

In mathematics, he was the original discoverer of the infinitesimal calculus. Newton's Philosophiae Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy, 1687) was one of the most important single works in the history of modern science.

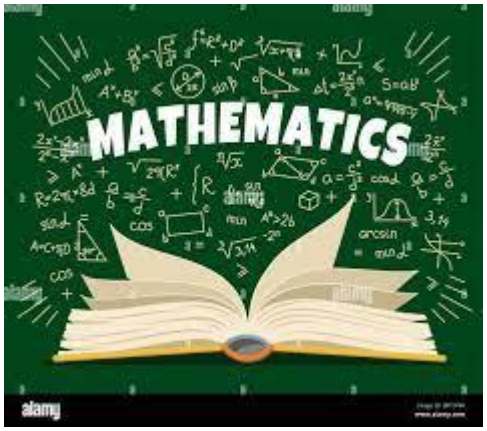
8. Answer the following questions for summary:



1) What does analysis incorporate?
2) Whose works does analysis builds on?
3) What do you know about Gottfried Wilhelm Leibniz?
4) What do you know about Isaac Newton?
5) What is numerical analysis?
6) What is called algorithm?
7) What is the name of the set of procedures the computer executes?
8) What is code?
9) What is numerical analysis concerned with?

Lesson 5
Mathematics theories

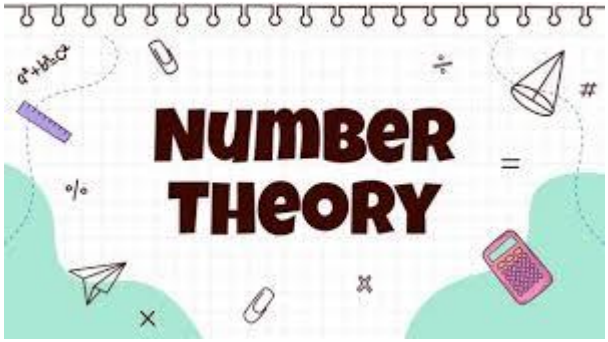
(From Encyclopedia Brittanica. Retrieved from: https://www.britannica.com/summary/analysis-mathematics)



Number theory is a branch of mathematics concerned with properties of and relations among integers. It is a popular subject among amateur mathematicians and students because of the wealth of seemingly simple problems that can be posed. Answers are much harder to come up with. It has been said that any unsolved mathematical problem of any interest more than a century old belongs to number theory. One of the best examples, recently solved, is Fermat's last theorem.

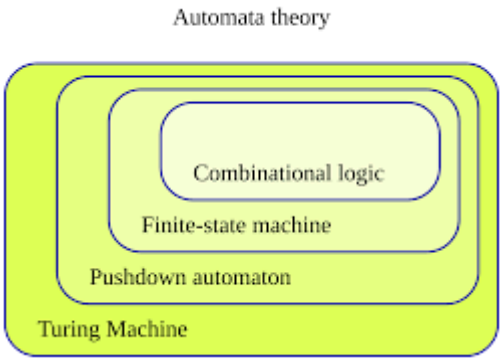
Automata theory is body of physical and logical principles underlying the operation of any electromechanical device (an automaton) that converts information input in one form into another, or into some action, according to an algorithm. Norbert Wiener and Alan M. Turing are regarded as pioneers in the field. In computer science, automata theory is concerned with the construction of robots from basic building blocks of automatons. The best example of a general automaton is an electronic digital computer. Networks of automata may be designed to mimic human behaviour.

1. Find the English equivalents in the text:



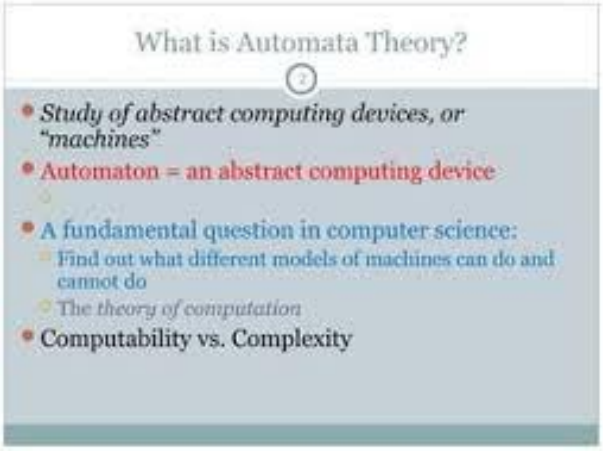
Теорія чисел, властивості, зв'язки, ціле число, математик-аматор, велика кількість простих завдань, придумати, нерозв'язана математична задача, належати, теорема Ферма, теорія автоматів, сукупність, лежати в основі роботи, електромеханічний пристрій, перетворювати введену інформацію, відповідно до алгоритму, вважатися піонерами, інформатика, конструювання роботів, бути розроблені, імітувати людську поведінку.

2. Translate the following terms and phrases:



Number theory, property, relations, integer, mathematical problem, Fermat's theorem, automata theory, logical principle, electromechanical device, automaton, to convert information input, algorithm, computer science, the construction of robots, building block, electronic digital computer, human behaviour.

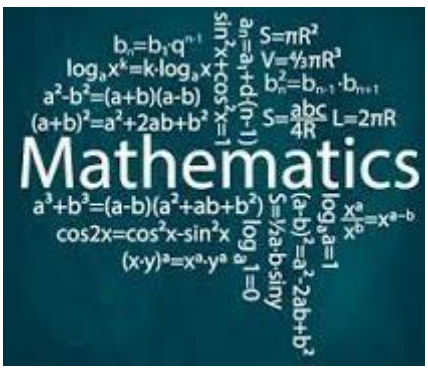
3. Match the terms (1-5) with the definitions (a-e):



1) Integer _____
2) Mathematical problem _____
3) Number theory _____
4) Automata theory_____
5) Robot_____

a) a problem that can be represented, analyzed, and possibly solved, with the methods of mathematics
b) a machine—especially one programmable by a computer—capable of carrying out a complex series of actions automatically.
c) the study of abstract machines and automata, as well as the computational problems that can be solved using them
d) a whole number (not a fractional number) that can be positive, negative, or zero
e) a branch of pure mathematics devoted primarily to the study of the integers and arithmetic functions

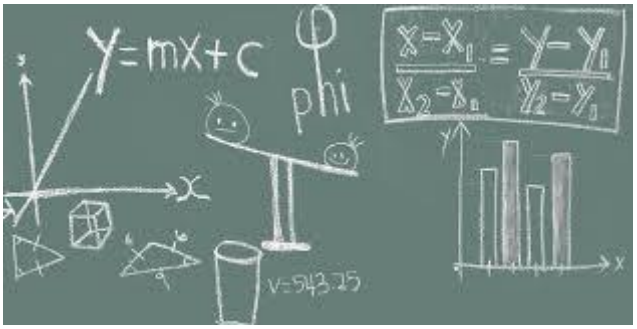
4. Complete the sentences using the following terms and phrases:



automata theory
robot
neutral integer
mathematical problem
number theory

1) _____is the study of the integers (e.g. whole numbers) and related objects.
2) involves solving the issue step by step, checking the solutions, and translating them back into the original context for testing and prediction.
3) _____deals with the logic of computation with respect to simple machines, referred to as automata.
4) A _____is a type of automated machine that can execute specific tasks with little or no human intervention and with speed and precision.
5) Zero is known as the_____**.**

5. Agree or disagree with the statements:



1) Automata theory is a branch of mathematics concerned with properties of and relations among integers. _____
2) Any unsolved mathematical problem of any interest more than a century old belongs to number theory. _____
3) One of the best examples of automata theory is Fermat's last theorem.
4) Number theory is body of physical and logical principles underlying the operation of any electromechanical device.
5) Fermat is regarded as a pioneer in the field of automata theory.
6) In computer science, automata theory is concerned with the construction of robots from basic building blocks of automatons.
7) The best example of a general automaton is an electronic digital computer.

6. Look at the slide and answer the questions:

1) What is mathematics called?
2) What is number theory called?
3) What is the role of integers?

NUMBER THEORY

Carl Friedrich Gauss, a great mathematician, once remarked that "*mathematics* is the **queen** of sciences, but *number theory* is the **queen** of mathematics". Number theory is the simplest of all types or branches of mathematics that even those without much mathematical training find it very interesting.

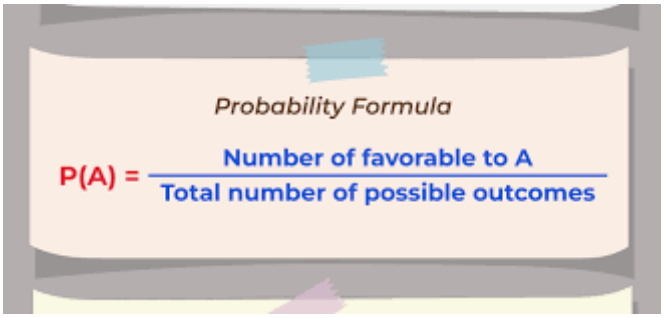
Properties of Integers

The set of integers (denoted by *Z*)

$$Z = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

plays a significant development of the concept of number. It possessed properties that developed mathematical ideas and expounded salient facts.

7. Read the text and put 4 questions to discuss in the group:

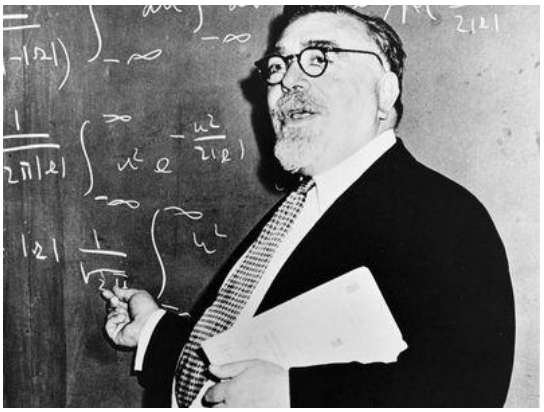


Probability theory is a branch of mathematics that deals with analysis of random events. Probability is the numerical assessment of likelihood on a scale from 0 (impossibility) to 1 (absolute certainty). Probability is usually expressed as the ratio between the number of ways an event can happen and the total number of things that can happen (e.g., there are 13 ways of picking a diamond from a deck of 52 cards, so the probability of picking a diamond is 13/52, or 1/4).

Probability theory grew out of attempts to understand card games and gambling. As science became more rigorous, analogies between certain biological, physical, and social phenomena and games of chance became more evident (e.g., the sexes of newborn infants follow sequences similar to those of coin tosses). As a result, probability became a fundamental tool of modern genetics and many other disciplines. Probability theory is also the basis of the insurance industry, in the form of actuarial statistics.

8. Read the text and put 3 questions to discuss them with the groupmates:

Norbert Wiener

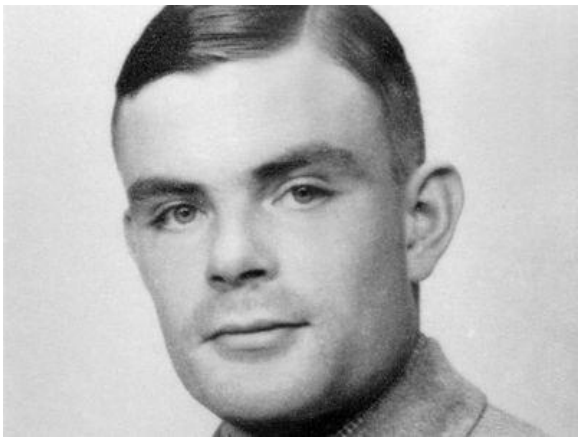


Norbert Wiener (born Nov. 26, 1894, Columbia, Mo., U.S.—died March 18, 1964, Stockholm, Swed.) was an American mathematician who established the science of cybernetics. He attained international renown by formulating some of the most important contributions to mathematics in the 20th century.

Wiener, a child prodigy whose education was controlled by his father, a professor of Slavonic languages and literature at Harvard University, graduated in mathematics from Tufts College (now Tufts University, Medford, Massachusetts) in 1909 at the age of 14. He spent a year at Harvard as a graduate student in zoology but left after he found that he was inept at laboratory work. At his father's suggestion, he began to study philosophy, and he completed a Ph.D. at Harvard in 1913 with a dissertation on mathematical logic.

9. Read the text and put 3 questions to discuss them with the groupmates:

Alan Turing



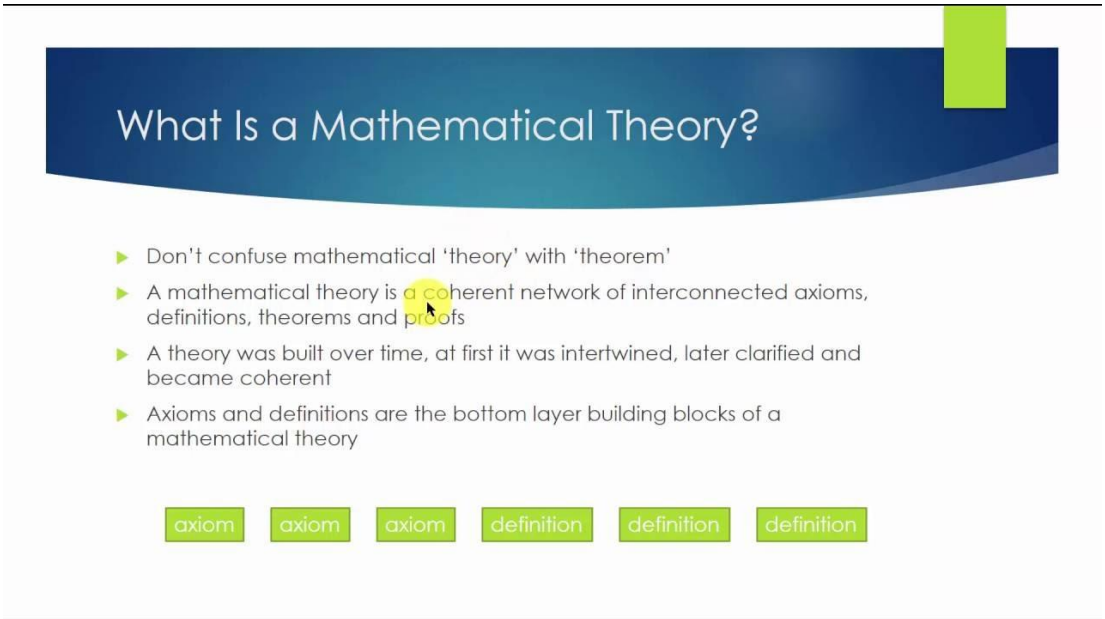
Alan Turing (born June 23, 1912, London, England—died June 7, 1954, Wilmslow, Cheshire) was a British mathematician and logician who made major contributions to mathematics, cryptanalysis, logic, philosophy, and mathematical biology and also to the new areas later named computer science, cognitive science, artificial intelligence, and artificial life.

The son of a civil servant, Turing was educated at a top private school. He entered the University of Cambridge to study mathematics in 1931. After graduating in 1934, he was elected to a fellowship at King's College (his college since 1931) in recognition of his research in probability theory.

His design for the Automatic Computing Engine (ACE) was the first complete specification of an electronic stored-program all-purpose digital computer.

Turing was a founding father of artificial intelligence and of modern cognitive science, and he was a leading early exponent of the hypothesis that the human brain is in large part a digital computing machine.

10.Answer the following questions to discuss in the group:



1) What is number theory concerned with?
2) Why is number theory a popular subject?
3) What is an example of unsolved mathematical problem?
4) What is automata theory concerned with?
5) Who is regarded as a pioneer in the field of automata theory?
6) What is the best example of a general automaton?
7) What do you know about Norbert Wiener?
8) What do you know about Alan Turing?

Glossary I

A

- abstract thoughts – абстрактні думки
- accounting – бухгалтерський облік
- adding – додавання
- algebra – алгебра
- algorithm – алгоритм
- analysis – аналіз
- applied math – прикладна математика
- applied mathematics – прикладна математика
- to approximate the processes of analysis - для наближення процесів аналізу
- area – область – площа
- arithmetic operations – арифметичні дії
- automata theory – теорія автоматів
- automaton – автомат
- axiom – аксіома
- axiomatic-deductive method – аксіоматично-дедуктивний метод

B

- branch of mathematics – галузь математики
- building block – будівельний блок
- business – бізнес

C

- the calculus of variations – варіаційне числення
- calculus - обчислення
- code – код
- complex analysis – комплексний аналіз
- complicated equations – складні рівняння
- computer programming – комп'ютерне програмування
- computer science – інформатика
- construction – будівництво
- the construction of robots – будівництво роботів
- continuity – безперервність
- to convert information input – конвертувати введену інформацію

D

- deductive logic – дедуктивна логіка
- derivative – похідна
- differentiability – диференційованість
- differential equations – диференціальні рівняння
- dividing – розділення – поділ

E

- electromechanical device – електромеханічний пристрій
- electronic digital computer – електронний цифровий комп’ютер
- engineering - інженерія
- equation - рівняння
- the error - помилка
- exponent – експонента – показник степеня

F

- Fermat’s theorem – теорема Ферма
- Fourier analysis – аналіз Фур’є
- functional analysis – функціональний аналіз

G

- geometric term – геометричний термін
- geometry – геометрія
- graph – графік

H

- human behaviour – поведінка людини

I

- inequality – нерівність
- infinite series – нескінченний ряд
- integer – ціле число
- the integral – інтеграл

L

- length – довжина
- limits – обмеження
- linear equation – лінійне рівняння
- logical principle – логічний принцип

M

- mathematical problem – математична задача
- mathematics – математика
- measurement – вимірювання
- multiplying – множення

N

- number theory – теорія чисел – вища арифметика
- number – номер
- numerical analysis – чисельний аналіз

O

- object – об’єкт
- often so complex that they require a computer – часто настільки складні, що потребують комп’ютера

P

- pattern – візерунок
- the perimeter of a regular polygon – периметр правильного багатокутника
- physical object – фізичний об’єкт
- physics – фізика
- plane geometry – плоска геометрія – планіметрія
- postulate – постулат
- program – програма
- property – власність

Q

- quadratic equation – квадратне рівняння

R

- real analysis – реальний аналіз
- relations – стосунки, відносини
- to represent a value – представляти значення

S

- science – наука
- shape – форма
- solid geometry – просторова геометрія – стереометрія
- spatial arrangement – просторове розташування
- spatial relationships – просторові відносини
- square root – квадратний корінь
- square – площа – квадрат
- structure – структура
- subfield – підполе
- subtracting – віднімання
- surrounding space – навколишній простір
- surveying – геодезія – топографічна зйомка

T

- tensor analysis – тензорний аналіз
- theorem – теорема

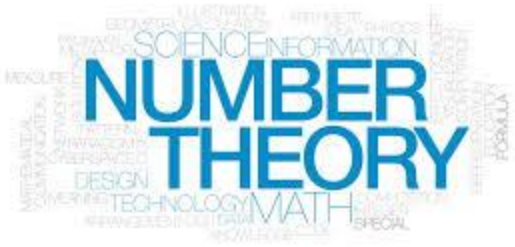
V

- variable – змінна
- vector analysis – векторний аналіз
- volume – обсяг

UNIT II
Number Theory

Lesson 1

Branch Definition and its History



Number Theory *(Alternative title: Higher Arithmetic)*
by William Dunham
(Encyclopedia Britannica. – Retrieved from: https://www.britannica.com/science/number-theory)

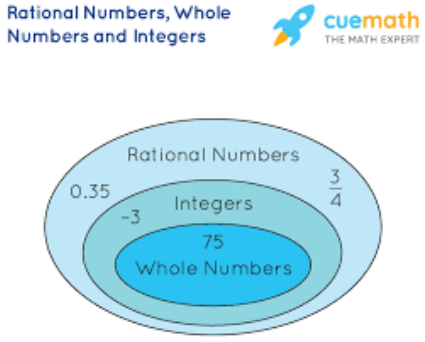
Number theory, branch of mathematics concerned with properties of the positive integers (1, 2, 3, …). Sometimes called "higher arithmetic," it is among the oldest and most natural of mathematical pursuits.

Number theory has always fascinated amateurs as well as professional mathematicians. In contrast to other branches of mathematics, many of the problems and theorems of number theory can be understood by laypersons, although solutions to the problems and proofs of the theorems often require a sophisticated mathematical background.

Until the mid-20th century, number theory was considered the purest branch of mathematics, with no direct applications to the real world. The advent of digital computers and digital communications revealed that number theory could provide unexpected answers to real-world problems. At the same time, improvements in computer technology enabled number theorists to make remarkable advances in factoring large numbers, determining primes, testing conjectures, and solving numerical problems once considered out of reach.

Modern number theory is a broad subject that is classified into subheadings such as elementary number theory, algebraic number theory, analytic number theory, geometric number theory, and probabilistic number theory. These categories reflect the methods used to address problems concerning the integers.

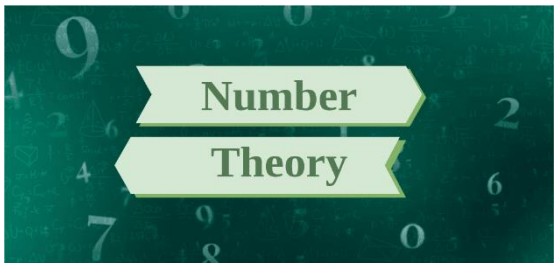
1. Find the English equivalents in the text:



Стосуватися (мати справу з), властивості чисел, математичні заняття, зачаровувати аматорів, на відміну від, бути зрозумілим для непрофесіоналів, вимагати складного математичного підґрунтя, виявити, удосконалення комп'ютерних технологій, здійснити великий прогрес, бути поза межами

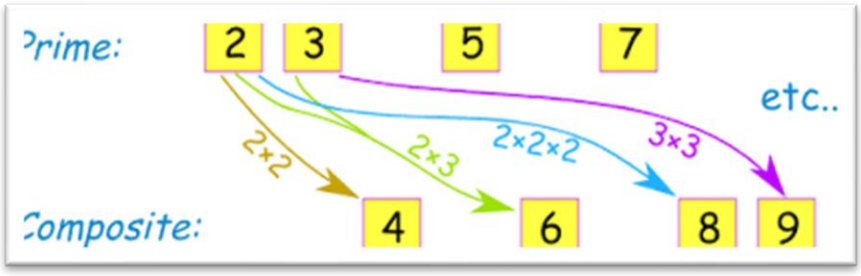
досяжності, класифікуватися на підрозділи (підзаголовки), відображати методи.

2. Translate the following terms and terminological expressions:



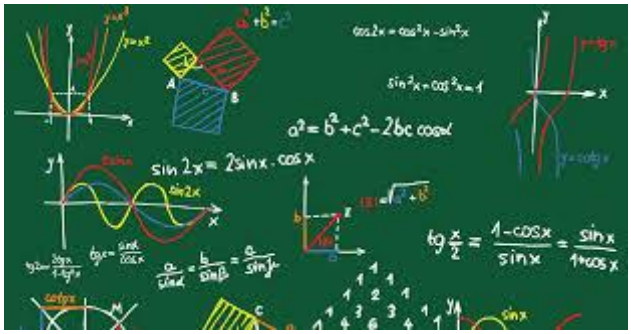
Number theory**,** branch of mathematics, positive integers, professional mathematician, proofs of the theorems, direct applications, digital computers, digital communications, factoring large numbers, determining primes, testing conjectures, solving numerical problems, elementary number theory, algebraic number theory, analytic number theory, geometric number theory, probabilistic number theory.

3. Say true or false:



1) Number theory is not considered to be a new branch of mathematics.
2) Number theory has always fascinated only professional mathematicians.
3) Many problems of number theory cannot be understood by laypersons.
4) Number theory has always been considered a branch of mathematics with direct applications to the real world.

4. Fill in the blanks:



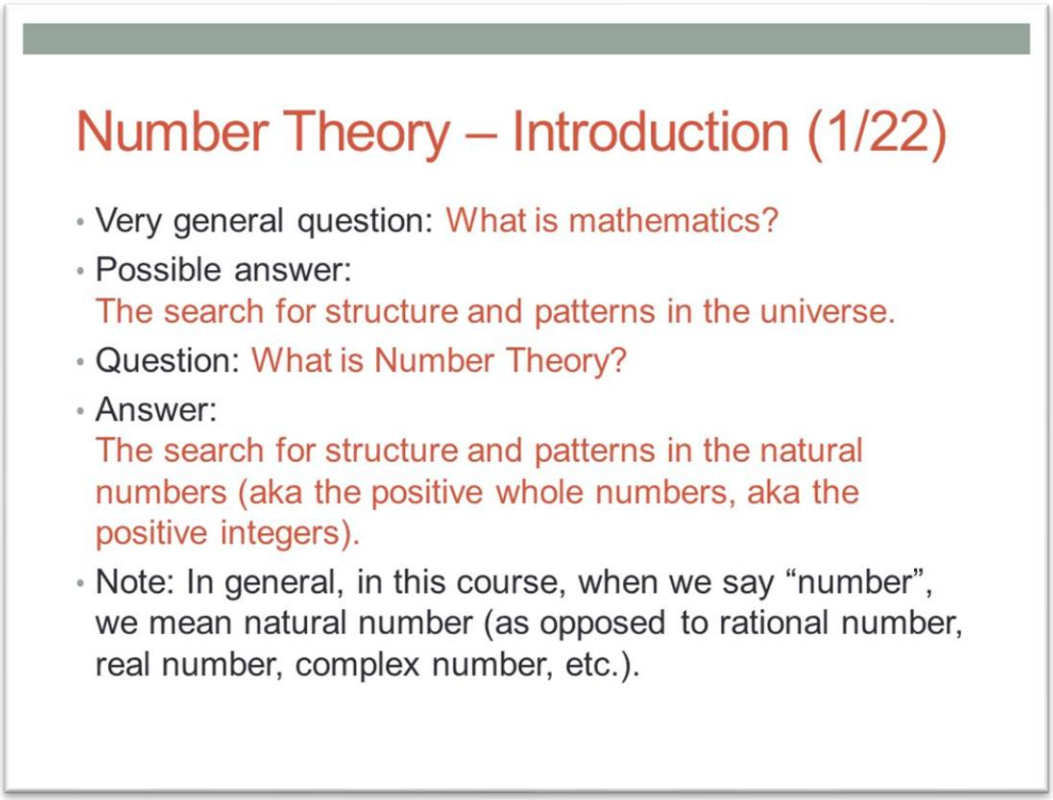
1) **Number theory** is a branch of mathematics concerned with properties of _____.
2) Solutions to the problems and proofs of the theorems often require a _____ background.
3) The advent of _____ _____ revealed that number theory could provide unexpected answers to real-world problems.

4) Modern number theory is a broad subject that is classified into subheadings such as _____.

5. Match the terms and their definitions:

1	Mathematics	the branch of mathematics concerned with the properties and relations of points, lines, surfaces, solids, and higher dimensional analogs	
2	Arithmetic	the study of such topics as quantity (number theory), structure (algebra), space (geometry), and change (*mathematical* analysis).	
3	Algebra	the branch of mathematics concerned with numerical calculations, such as addition, subtraction, multiplication, and division	
4	Geometry	the study of mathematical symbols and the rules for manipulating these symbols	

6. Read and translate the information in the slide:



7. Read the text, put 3 questions to it to discuss them in the group:

Carl Friedrich Gauss

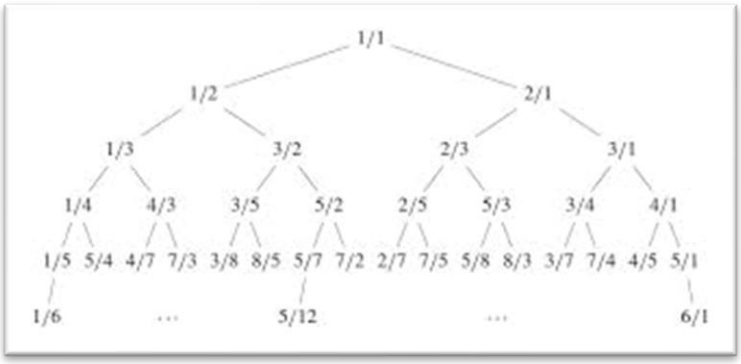
(from Gray J.J. Carl Friedrich Gauss. German mathematician. Encyclopedia Britannica. Retrieved from: https://www.britannica.com/biography/Carl-Friedrich-Gauss)

Carl Friedrich Gauss (born April 30, 1777, Brunswick, Germany ─ died February 23, 1855, Göttingen, Hanover) was a German mathematician, generally regarded as one of the greatest mathematicians of all time for his contributions to number theory, geometry, probability theory, geodesy, planetary astronomy, the theory of functions, and potential theory (including electromagnetism).

Gauss was the only child of poor parents. He was rare among mathematicians in that he was a calculating prodigy, and he retained the ability to do elaborate calculations in his head most of his life. Impressed by this ability and by his gift for languages, his teachers and his devoted mother recommended him to the duke of Brunswick in 1791, who granted him financial assistance to continue his education locally and then to study mathematics at the University of Göttingen from 1795 to 1798.

Gauss's pioneering work gradually established him as the era's preeminent mathematician, first in the German-speaking world and then farther afield.

8. Answer the following questions for summary:

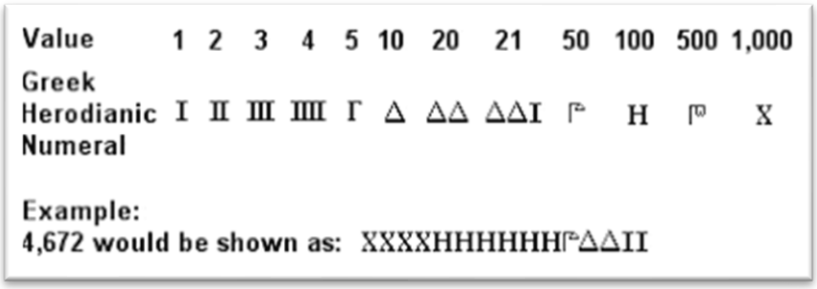


1) What is number theory concerned with?
2) What branch was number theory considered until the mid-20th century?
3) What did the advent of digital computers and digital communications reveal?
4) What does modern number theory involve?
5) Why is Carl Friedrich Gauss famous?

Lesson 2

From Prehistory Through Classical Greece

(*by the Editors of the Encyclopedia Britannica.*
Last Updated: Oct 30, 2024 • Article History. Retrieved from:
https://www.britannica.com/science/number-theory)

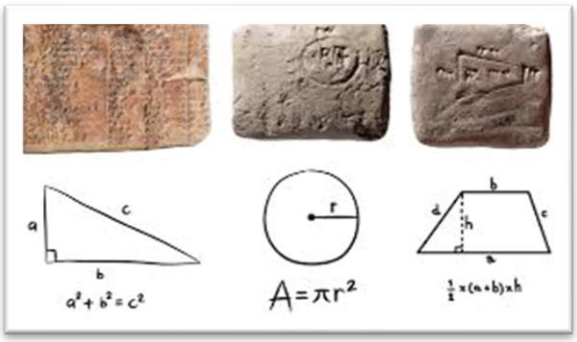


The ability to count dates back to prehistoric times. This is evident from archaeological artifacts, such as a 10,000-year-old bone from the Congo region of Africa with tally marks scratched upon it—signs of an unknown ancestor counting something. Very near the dawn of civilization, people had grasped the idea of "multiplicity" and thereby had taken the first steps toward a study of numbers.

It is certain that an understanding of numbers existed in ancient Mesopotamia, Egypt, China, and India, for tablets, papyri, and temple carvings from these early cultures have survived. A Babylonian tablet known as Plimpton 322 (*c.* 1700 BC) is a case in point. In modern notation, it displays number triples *x*, *y*, and *z* with the property that $x^2 + y^2 = z^2$. One such triple is 2,291, 2,700, and 3,541, where $2{,}291^2 + 2{,}700^2 = 3{,}541^2$. This certainly reveals a degree of number theoretic sophistication in ancient Babylon.

Despite such isolated results, a general theory of numbers was nonexistent. For this—as with so much of theoretical mathematics—one must look to the Classical Greeks, whose groundbreaking achievements displayed an odd fusion of the mystical tendencies of the Pythagoreans and the severe logic of Euclid's *Elements* (*c.* 300 BC).

1. Find the English equivalents in the text:



Бути очевидним, позначки підрахунків, невідомий предок, зоря цивілізації, осягнути ідею, храмове різьблення, конкретний випадок, теоретична витонченість, новаторські досягнення, дивне злиття, сувора логіка.

2. Translate the following terms and terminological expressions:

The ability to count, archaeological artifacts, sign, multiplicity, tablets, modern notation, property, theoretical mathematics.

3. Say true or false:

1) Archaeological artifacts did not show the ability to count in prehistoric times.
2) People took the first steps toward a study of numbers in ancient times.
3) General theory of numbers existed in ancient Mesopotamia.
4) General theory of numbers existed in ancient Greece.

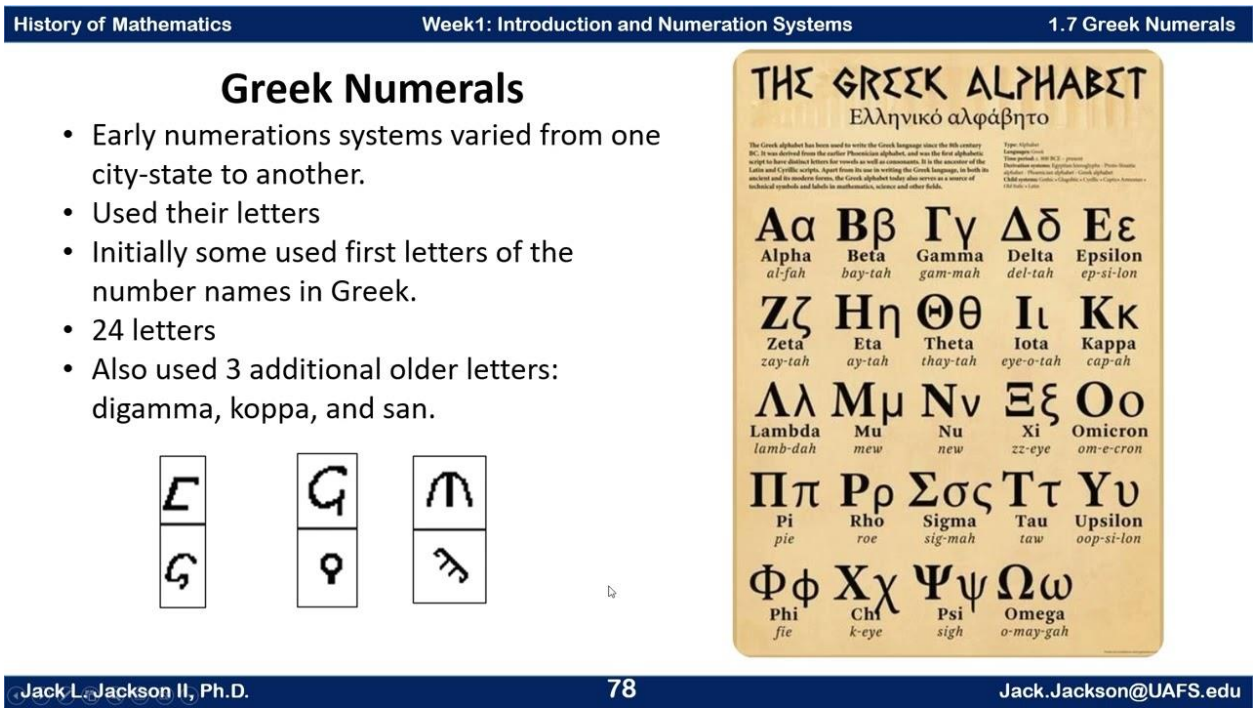
4. Fill in the blanks:

1) An ancient bone found in Africa shows signs of _____.
2) In modern notation, a Babylonian tablet displays _____.
3) A general theory of numbers was nonexistent despite _____.
4) Classical Greeks had _____ achievements.

5. Find the definitions of the notions:

1	Counting	the number of times in the multiset	
2	Multiplicity	an attribute, quality, or characteristic of something	
3	Triple	determining the total number	
4	Property	made up of three standard units or items	

6. Look at the slide and name the features of the ancient Greek numbers:



7. Read the text, put 3 questions to it to discuss them in the group:

Pierre de Fermat

(from Carl B. Boyer. Pierre de Fermat.French mathematician. Encyclopedia Britannica. Retrieved from: https://www.britannica.com/biography/Pierre-de-Fermat)

Pierre de Fermat (born August 17, 1601, Beaumont-de-Lomagne, France—died January 12, 1665, Castres) was a French mathematician who is often called the founder of the modern theory of numbers. Together with René Descartes, Fermat was one of the two leading mathematicians of the first half of the 17th century. Independently of Descartes, Fermat discovered the fundamental principle of analytic geometry. His methods for finding tangents to curves and their maximum and minimum points led him to be regarded as the inventor of the differential calculus. Through his correspondence with Blaise Pascal he was a co-founder of the theory of probability.

Little is known of Fermat's early life and education. He was of Basque origin and received his primary education in a local Franciscan school. He studied law, probably at Toulouse and perhaps also at Bordeaux. Having developed tastes

for foreign languages, classical literature, and ancient science and mathematics, Fermat followed the custom of his day in composing conjectural “restorations” of lost works of antiquity. By 1629 he had begun a reconstruction of the long-lost Plane Loci of Apollonius, the Greek geometer of the 3rd century bce. He soon found that the study of loci, or sets of points with certain characteristics, could be facilitated by the application of algebra to geometry through a coordinate system. Meanwhile, Descartes had observed the same basic principle of analytic geometry, that equations in two variable quantities define plane curves. Because Fermat’s Introduction to Loci was published posthumously in 1679, the exploitation of their discovery, initiated in Descartes’s Géométrie of 1637, has since been known as Cartesian geometry.

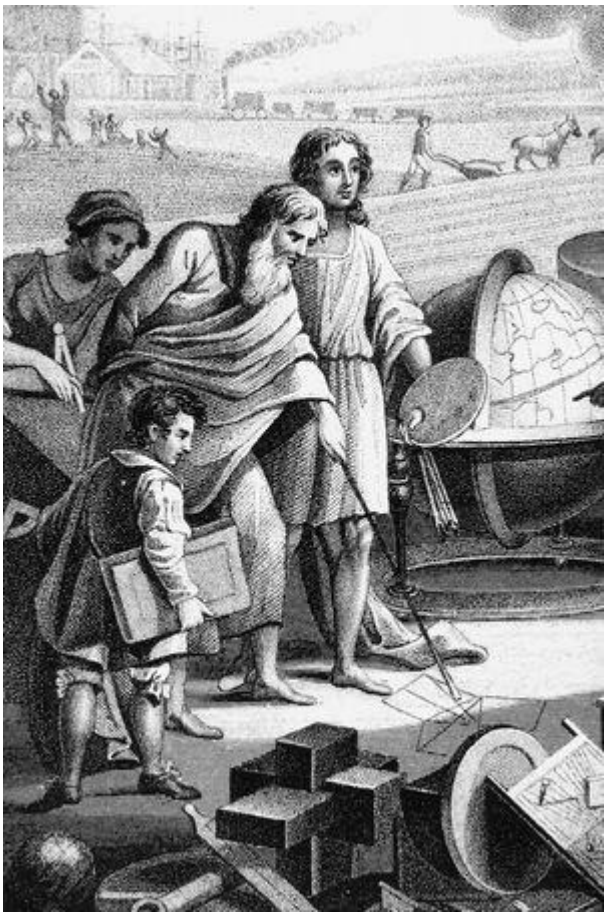
8. Answer the following questions for summary:

1) When do the ability to count date back to?
2) When did people take the first steps toward a study of numbers?
3) What do tablets, papyri, and temple carvings show?
4) Did a general theory of numbers exist in ancient world?
5) Who is often called the founder of the modern theory of numbers?
6) What do you know about Pierre de Fermat?

Lesson 3

Pythagoras

(*by the Editors of the Encyclopedia Britannica.*
Pythagoras. Retrieved from: https://www.britannica.com/biography/Pythagoras)



According to tradition, Pythagoras (*c.* 580–500 BC) worked in southern Italy amid devoted followers. His philosophy enshrined number as the unifying concept necessary for understanding everything from planetary motion to musical harmony. Given this viewpoint, it is not surprising that the Pythagoreans attributed quasi-rational properties to certain numbers.

For instance, they attached significance to perfect numbers—i.e., those that equal the sum of their proper divisors. Examples are 6 (whose proper divisors 1, 2, and 3 sum to 6) and 28 (1 + 2 + 4 + 7 + 14). The Greek philosopher Nicomachus of Gerasa (flourished *c.* AD 100), writing centuries after Pythagoras but clearly in his philosophical debt, stated that perfect numbers represented “virtues, wealth, moderation, propriety, and beauty.” (Some modern writers label such nonsense numerical theology.)

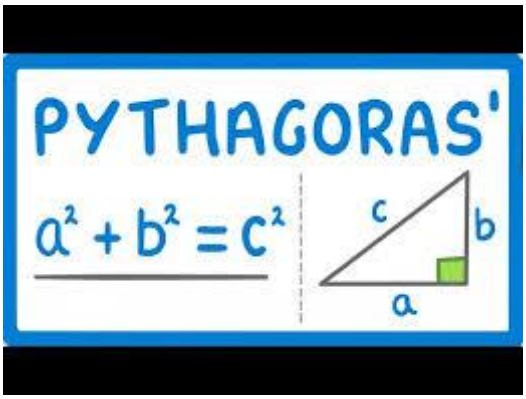
In a similar vein, the Greeks called a pair of integers amicable (“friendly”) if each was the sum of the proper divisors of the other. They knew only a single amicable pair: 220 and 284. One can easily check that the sum of the proper divisors of 284 is 1 + 2 + 4 + 71 + 142 = 220 and the sum of the proper divisors of 220 is 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284. For those prone to number mysticism, such a phenomenon must have seemed like magic.

1. Find the English equivalents in the text:

Відданий послідовник, містити число, планетарний рух, точка зору, приписувати властивості, надавати значення, заявляти, чесноти, багатство, помірність, пристойність, краса, подібним чином, легко перевірити, бути схильним.

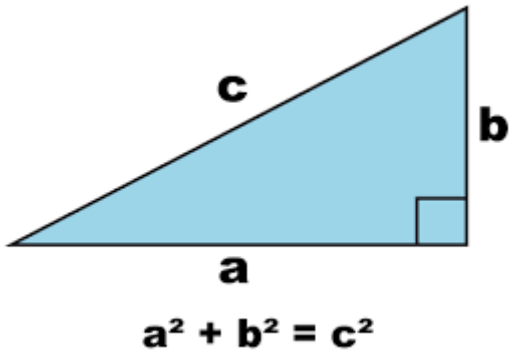
2. **Translate the following terms and terminological expressions:**
Quasi-rational properties, certain numbers, perfect numbers, proper divisors, amicable pair of integers, the sum of the proper divisors, number mysticism.

3. **Say true or false:**



1) Pythagoras is not known to have worked among his followers.
2) Pythagoras is believed to have considered number as the unifying concept.
3) Pythagoras and his followers gave importance to perfect numbers.
4) Pythagoras stated that perfect numbers represented "virtues, wealth, moderation, propriety, and beauty".

4. **Fill in the blanks:**

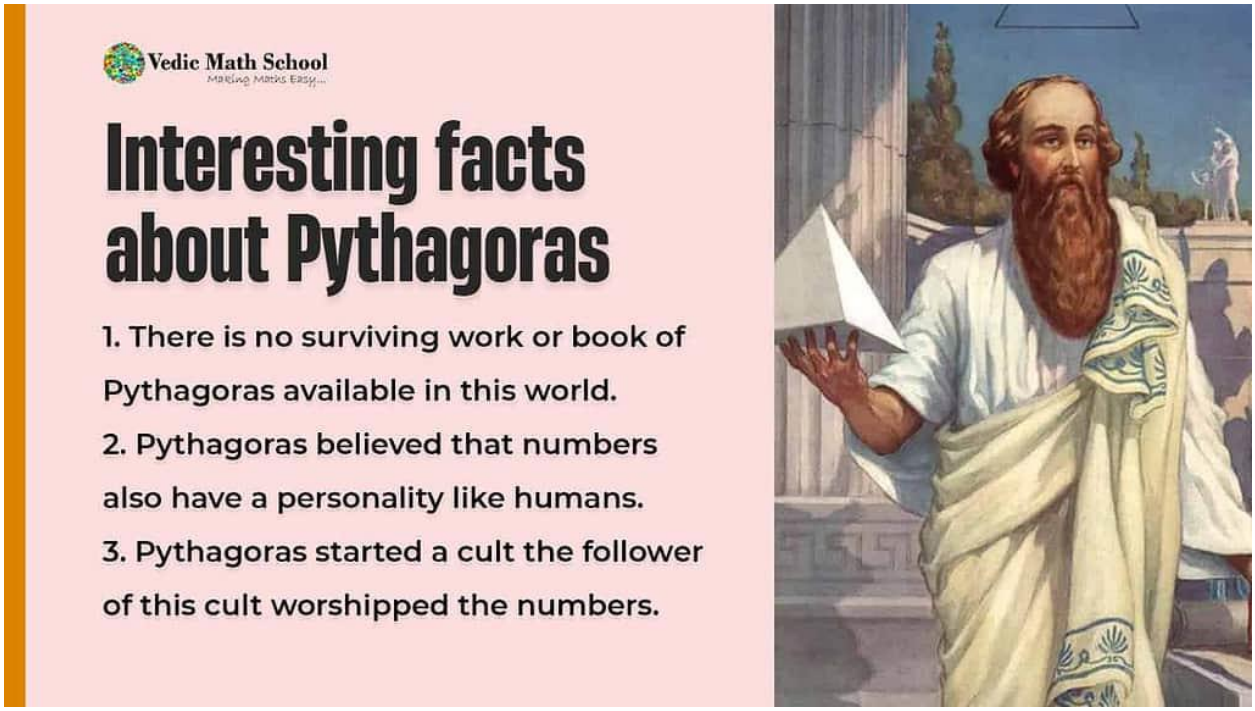


1) Perfect numbers are numbers that .....
2) The examples of perfect numbers are .....
3) A pair of integers was amicable if each was .....

5. **Match the terms and their definitions:**

1	Number	a number by which another number is to be divided	
2	Sum	the total amount resulting from the addition of two or more numbers, amounts, or items	
3	Divisor	a whole number; a number that is not a fraction	
4	Integer	an arithmetical value, expressed by a word, symbol, or figure, representing a particular quantity and used in counting and making calculations and for showing order in a series or for identification	

6. **Look at the slide and say what facts about Pythagoras you know:**



7. **Read the text about Pythagoras, put 5 questions and ask your friends to answer them.**

Pythagoras

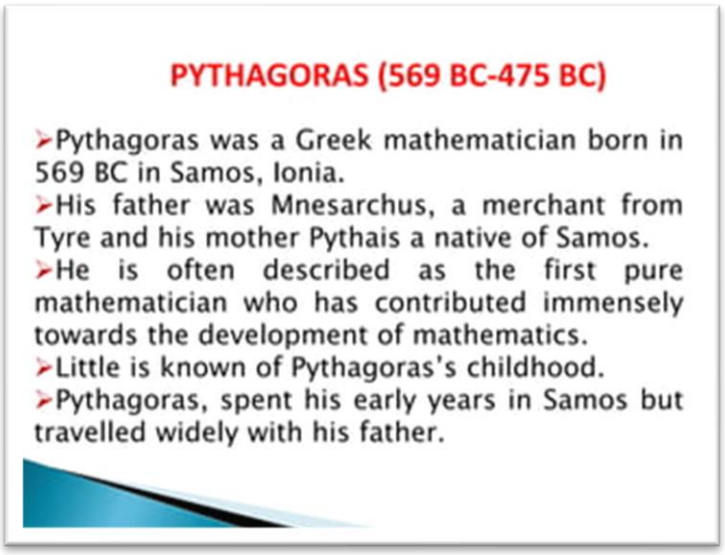
by Adam Augustyn

Pythagoras, (born *c* 570 BCE, Greece—died *c.* 500–490 BCE, Italy) was a Greek philosopher, mathematician, and founder of the Pythagorean brotherhood. Although religious in nature, the Pythagorean brotherhood formulated principles that influenced the thought of Plato and Aristotle and contributed to the development of mathematics and Western rational philosophy.

Pythagoras emigrated to southern Italy about 532 BCE, apparently to escape Samos′s tyrannical rule, and established his ethico-political academy at Croton (now Crotone, Italy). It is difficult to distinguish Pythagoras′s teachings from those of his disciples. None of his writings have survived, and Pythagoreans invariably supported their doctrines by indiscriminately citing their master′s authority. Pythagoras, however, is generally credited with the theory of the functional significance of numbers in the objective world and in music. Other

discoveries often attributed to him (the incommensurability of the side and diagonal of a square, for example, and the Pythagorean theorem for right triangles) were probably developed only later by the Pythagorean school. More probably, the bulk of the intellectual tradition originating with Pythagoras himself belongs to mystical wisdom rather than to scientific scholarship.

8. Answer the following questions:

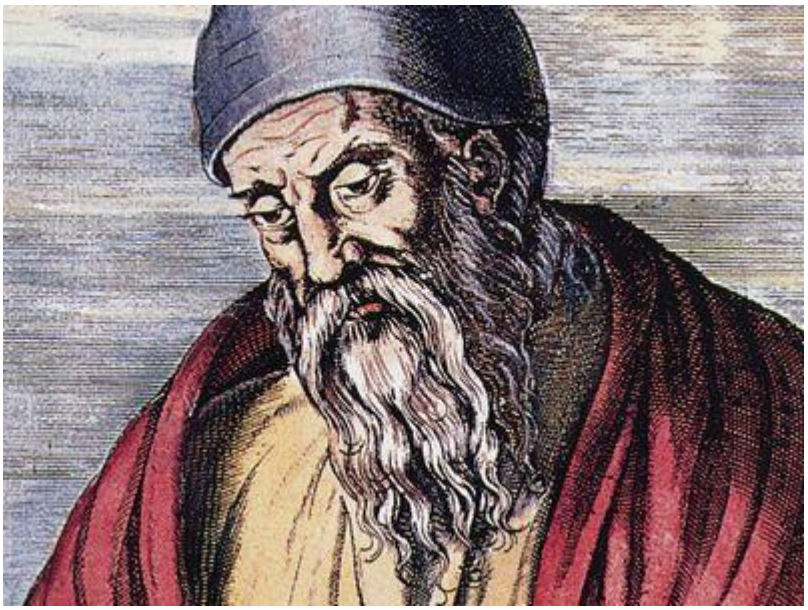


1) How did Pythagoras consider number?
2) What numbers did Pythagoras attach significance?
3) What meaning did Nicomachus of Gerasa attribute to number?
4) What is the definition of amicable pair of integers according to the Greeks?
5) What was Pythagoras's profession?
6) What was Pythagoras known for?

Lesson 4

Euclid

(From Taisbak C.M. Euclid. Greek Mathematician. Encyclopedia Britannica. Retrieved from: https://www.britannica.com/biography/Euclid-Greek-mathematician)



Euclid began Book VII of his *Elements* by defining a number as "a multitude composed of units." The plural here excluded 1; for Euclid, 2 was the smallest "number." He later defined a prime as a number "measured by a unit alone" (i.e., whose only proper divisor is 1), a composite as a number that is not prime, and a perfect number as one that equals the sum of its "parts" (i.e., its proper divisors).

From there, Euclid proved a sequence of theorems that marks the beginning of number theory as a mathematical (as opposed to a numerological) enterprise. Four Euclidean propositions deserve special mention.

The first, Proposition 2 of Book VII, is a procedure for finding the greatest common divisor of two whole numbers. This fundamental result is now called the Euclidean algorithm in his honour.

Second, Euclid gave a version of what is known as the unique factorization theorem or the fundamental theorem of arithmetic. This says that any whole number can be factored into the product of primes in one and only one way. For example, 1,960 = 2 × 2 × 2 × 5 × 7 × 7 is a decomposition into prime factors, and no other such decomposition exists. Euclid's discussion of unique factorization is not satisfactory by modern standards, but its essence can be found in Proposition 32 of Book VII and Proposition 14 of Book IX.

Third, Euclid showed that no finite collection of primes contains them all. His argument, Proposition 20 of Book IX, remains one of the most elegant proofs in all of mathematics. Beginning with any finite collection of primes—say, a, b, c, ..., n—Euclid considered the number formed by adding one to their product: $N = (abc\cdots n) + 1$.

Fourth, Euclid ended Book IX with a blockbuster: if the series $1 + 2 + 4 + 8 + \ldots + 2^k$ sums to a prime, then the number $N = 2^k(1 + 2 + 4 + \ldots + 2^k)$ must be perfect. For example, $1 + 2 + 4 = 7$, a prime, so $4(1 + 2 + 4) = 28$ is perfect.

Euclid’s “recipe” for perfect numbers was a most impressive achievement for its day.

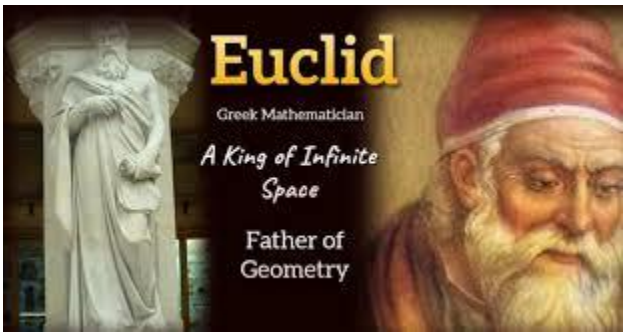
1.Find the English equivalents in the text:

Визначити число (дати визначення числу), множина, виключати, бути обмеженим, довести послідовність теорем, заслуговувати особливої уваги, назвати на його честь, бути задовільним, вражаюче досягнення.

2.Translate the following terms and terminological expressions:

Multitude, plural, proper divisor, the greatest common divisor of two whole numbers, the Euclidean algorithm, the unique factorization theorem, the fundamental theorem of arithmetic, decomposition into prime factors, finite collection of primes, perfect number.

3. Read the text fragment and underline mathematical terms:



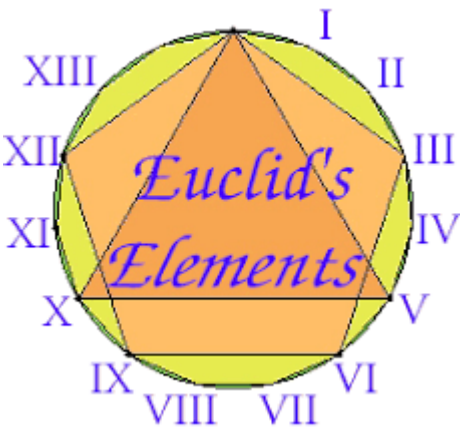
Euclid examined the two alternatives:

(1) If N is prime, then it is a new prime not among $a, b, c, \ldots, n$ because it is larger than all of these. For example, if the original primes were 2, 3, and 7, then $N = (2 \times 3 \times 7) + 1 = 43$ is a larger prime.

(2) Alternately, if N is composite, it must have a prime factor which, as Euclid demonstrated, cannot be one of the originals. To illustrate, begin with primes 2, 7, and 11, so that $N = (2 \times 7 \times 11) + 1 = 155$.

This is composite, but its prime factors 5 and 31 do not appear among the originals. Either way, a finite set of primes can always be augmented. It follows, by this beautiful piece of logic, that the collection of primes is infinite.

4. Read the article and put 5 questions for a discussion with the groupmates:



Euclid and his Elements

(From Taisbak C.M. Euclid. Greek Mathematician. Encyclopedia Britannica. Retrieved from: https://www.britannica.com/biography/Euclid-Greek-mathematician)

Euclid (300 BCE, Alexandria, Egypt) was the most prominent mathematician of Greco-Roman antiquity.

Euclid compiled his *Elements* from a number of works of earlier men.

Book I then proves elementary theorems about triangles and parallelograms and ends with the Pythagorean theorem.

The subject of Book II has been called geometric algebra because it states algebraic identities as theorems about equivalent geometric figures.

Book III deals with properties of circles and Book IV with the construction of regular polygons, in particular the pentagon.

Book V shifts from plane geometry to expound a general theory of ratios and proportions that is attributed by Proclus (along with Book XII) to Eudoxus of Cnidus.

Book VI applies this theory of ratios to plane geometry, mainly triangles and parallelograms, culminating in the "application of areas," a procedure for solving quadratic problems by geometric means.

Books VII–IX contain elements of number theory, where number (*arithmos*) means positive integers greater than 1.

Book VIII examines numbers in continued proportions, now known as geometric sequences (such as ax, ax^2, ax^3, ax^4...); and Book IX proves that there are an infinite number of primes.

According to Proclus, Books X and XIII incorporate the work of the Pythagorean Theaetetus (*c.* 417–369 BCE). Book X, which comprises roughly one-fourth of the *Elements*, seems disproportionate to the importance of its classification of incommensurable lines and areas (although study of this book would inspire Johannes Kepler [1571–1630] in his search for a cosmological model).

Books XI–XIII examine three-dimensional figures, in Greek *stereometria*. Book XI concerns the intersections of planes, lines, and parallelepipeds (solids with parallel parallelograms as opposite faces).

Book XII applies Eudoxus's method of exhaustion to prove that the areas of circles are to one another as the squares of their diameters and that the volumes of spheres are to one another as the cubes of their diameters.

Book XIII culminates with the construction of the five regular Platonic solids (pyramid, cube, octahedron, dodecahedron, icosahedron) in a given sphere, as displayed in the animation.

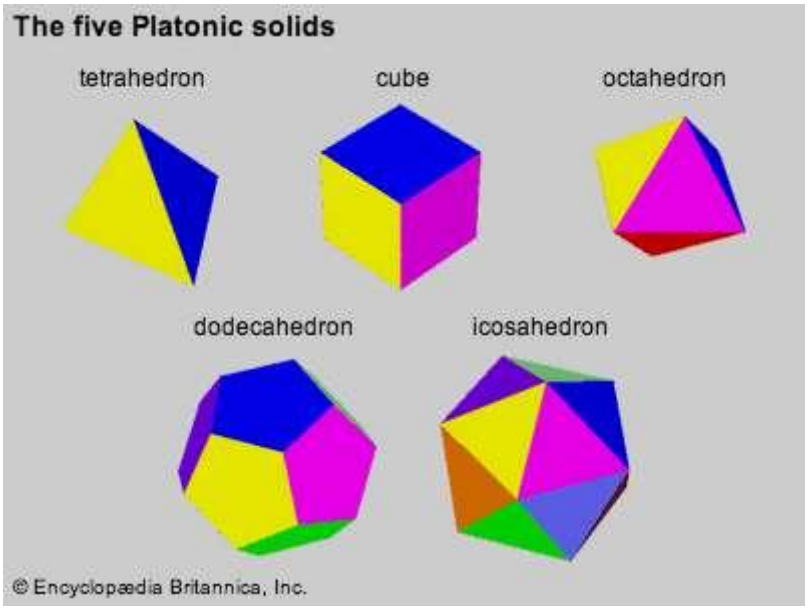
Euclid’s contemporaries considered his work final and authoritative; if more was to be said, it had to be as commentaries to the *Elements*.

5. **Read and translate five unproved assumptions that Euclid called postulates (now known as axioms), and five further unproved assumptions that he called common notions.**

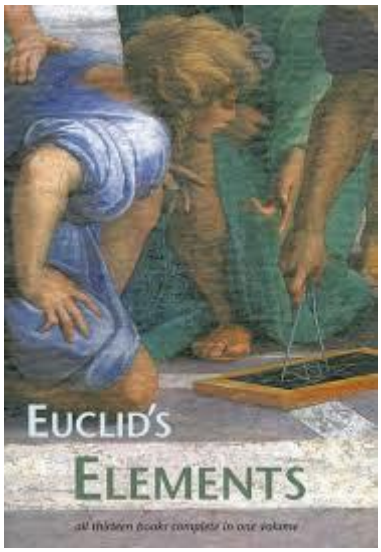
	Euclid's axioms
1	Given two points there is one straight line that joins them.
2	A straight line segment can be prolonged indefinitely.
3	A circle can be constructed when a point for its centre and a distance for its radius are given.
4	All right angles are equal.
5	If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles.
	Euclid's common notions
6	Things equal to the same thing are equal.
7	If equals are added to equals, the wholes are equal.
8	If equals are subtracted from equals, the remainders are equal.
9	Things that coincide with one another are equal.
10	The whole is greater than a part.

6. **Look at "The five Platonic solids" and make your drawings in your notebooks:**

These are the only geometric solids whose faces are composed of regular, identical polygons (*Encyclopædia Britannica, Inc.)*



7. **Answer the questions for summary:**



1) How did Euclid begin Book VII of his *Elements?*
2) How did Euclid define a prime?
3) What marks the beginning of number theory as a mathematical enterprise?
4) What is now called the Euclidean algorithm?
5) What was a most impressive achievement for that day?
6) How did Euclid become famous?
7) Where was Euclid from?

Lesson 5

Diophantus

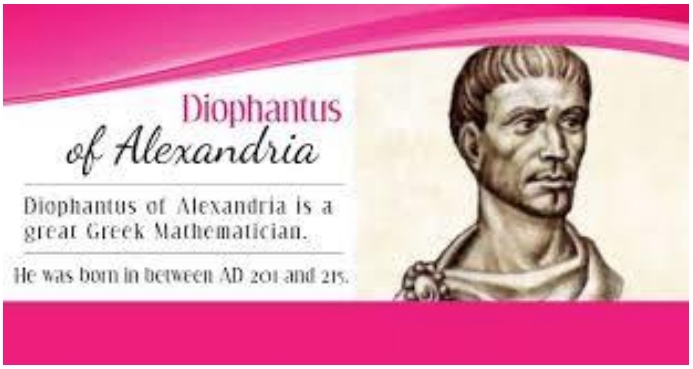
(From Sesiano J. Diophantus of Alexandria. Encyclopedia Britannica. Retrieved from: https://www.britannica.com/biography/Diophantus)

Diophantus (flourished c. CE 250) was a Greek mathematician, famous for his work in algebra.

What little is known of Diophantus's life is circumstantial. From the appellation "of Alexandria" it seems that he worked in the main scientific centre of the ancient Greek world; and because he is not mentioned before the 4th century, it seems likely that he flourished during the 3rd century. An arithmetic epigram from the Anthologia Graeca of late antiquity, purported to retrace some landmarks of his life (marriage at 33, birth of his son at 38, death of his son four years before his own at 84), may well be contrived.

Two works have come down to us under his name, both incomplete. The first is a small fragment on polygonal numbers (a number is polygonal if that same number of dots can be arranged in the form of a regular polygon). The second, a large and extremely influential treatise upon which all the ancient and modern fame of Diophantus reposes, is his Arithmetica. Its historical importance is twofold: it is the first known work to employ algebra in a modern style, and it inspired the rebirth of number theory.

1. Find the English equivalents in the text:



Бути відомим чимось, головний науковий центр, згадуватися, досягти розквіту, арифметична епіграма, мати на меті, відстежити орієнтири, бути надуманим, незавершений твір, багатокутне число, впливовий трактат, стародавня слава, подвійне значення, застосовувати алгебру, надихати, відродження.

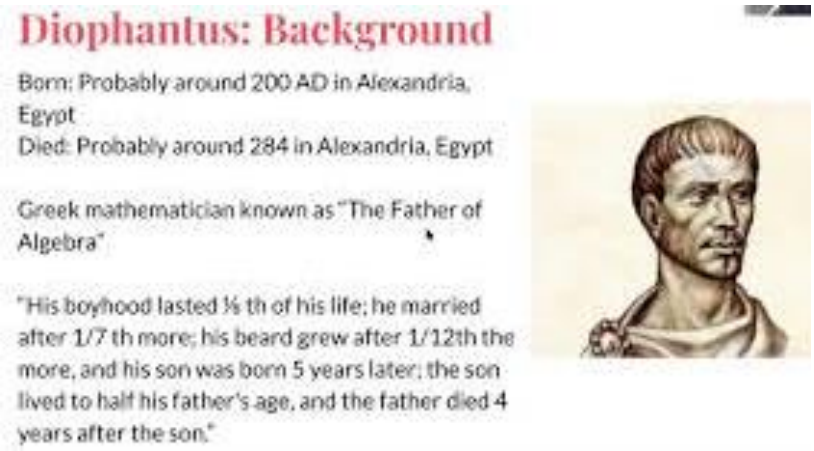
2. Translate the terms and phrases:

Mathematician, algebra, main scientific centre, the ancient Greek world, arithmetic epigram, to retrace some landmarks, polygonal numbers, dot, regular polygon, treatise, to employ algebra, number theory.

3. Match the terms and their definitions:

1	polygon	a branch of pure mathematics devoted primarily to the study of the integers and arithmetic functions	
2	polygonal number	the branch of mathematics that helps in the representation of problems or situations in the form of mathematical expressions	
3	regular polygon	a number that counts dots arranged in the shape of a regular polygon	
4	algebra	a polygon that is direct equiangular (all angles are equal in measure) and equilateral (all sides have the same length)	
5	number theory	a plane figure made up of line segments connected to form a closed polygonal chain	

4. Complete the sentences with the following terms and phrases:

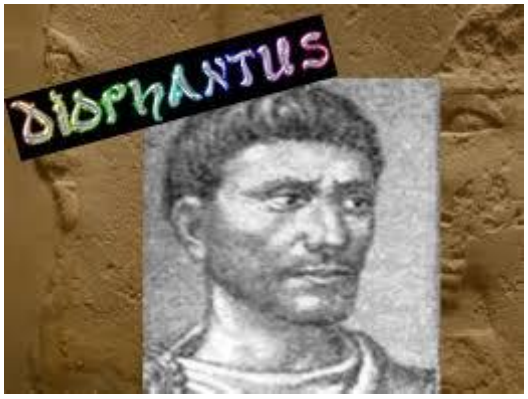


algebra
number theory
polygon
polygonal numbers
regular polygons

1) If all the sides and interior angles of the polygons are equal, they are known as_____.

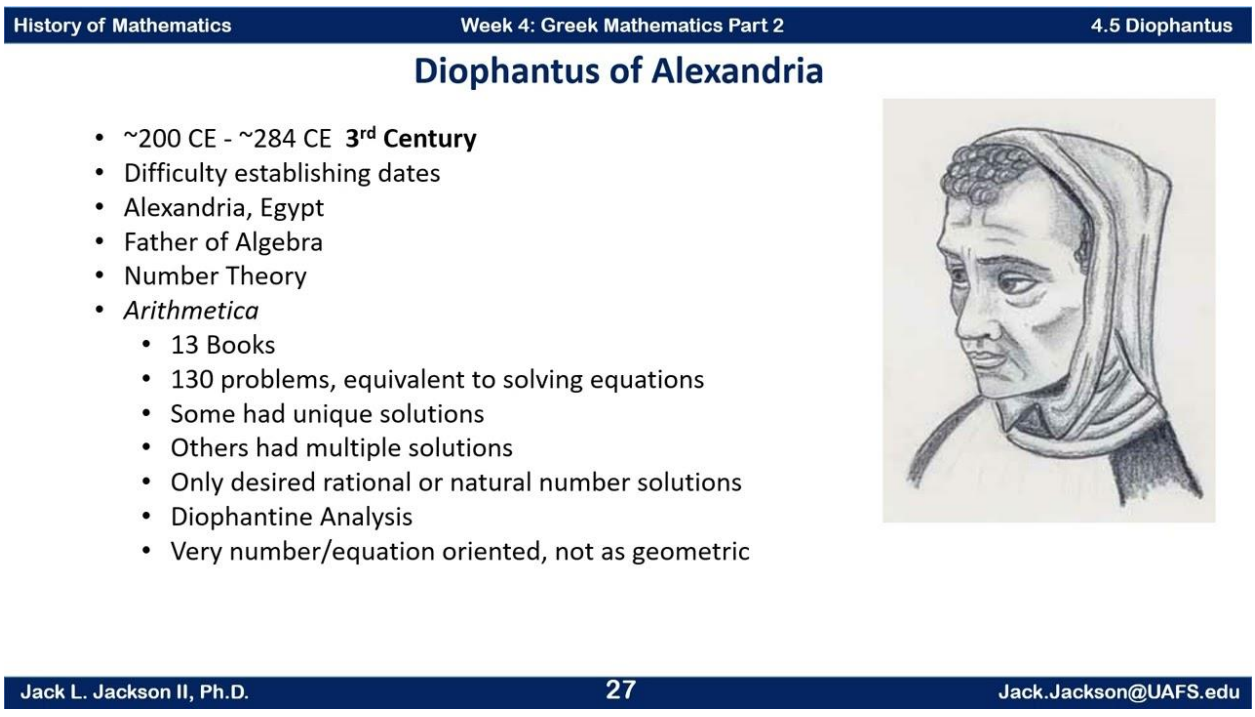
2) _____involves variables like x, y, z, and mathematical operations like addition, subtraction, multiplication, and division to form a meaningful mathematical expression.
3) A _____is a closed figure on a plane formed from a finite number of lines segments connected end-to-end.
4) _____helps to study the relationships between different sorts of numbers.
5) The concept of _____was first defined by the Greek mathematician Hypsicles in the year 170 BC (Heath 126).

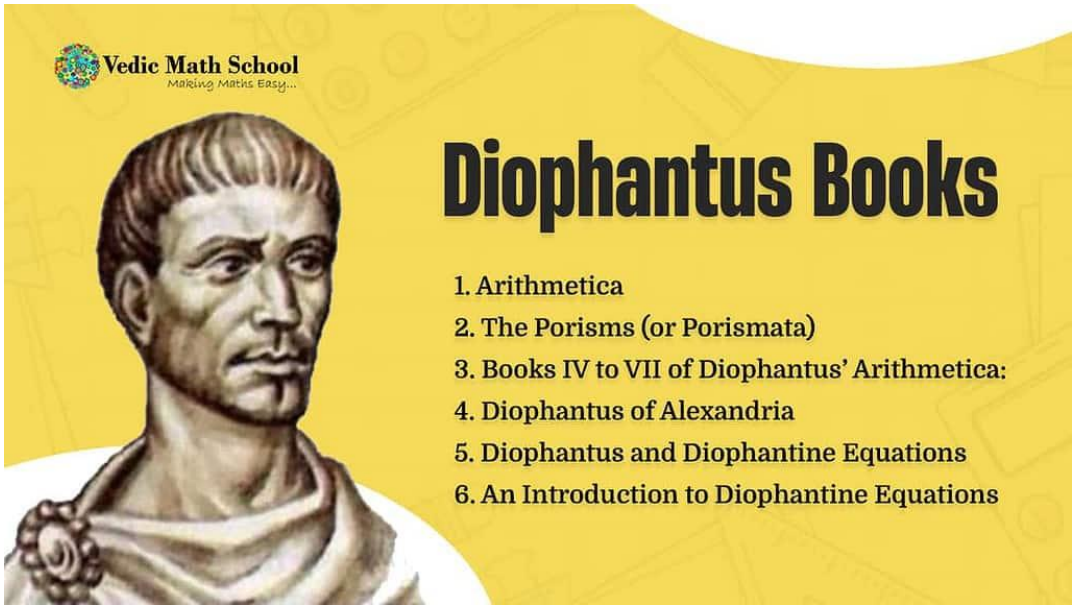
5. Agree or disagree with the statements:



1) Diophantus was a Greek mathematician, famous for his work in geometry.
2) Diophantus flourished during the 4th century.
3) His contribution is known from his three books.
4) His first work is a small fragment on polygonal numbers.
5) His second work, a large and extremely influential treatise, is his Arithmetica.
6) His Arithmetica has great historical importance.

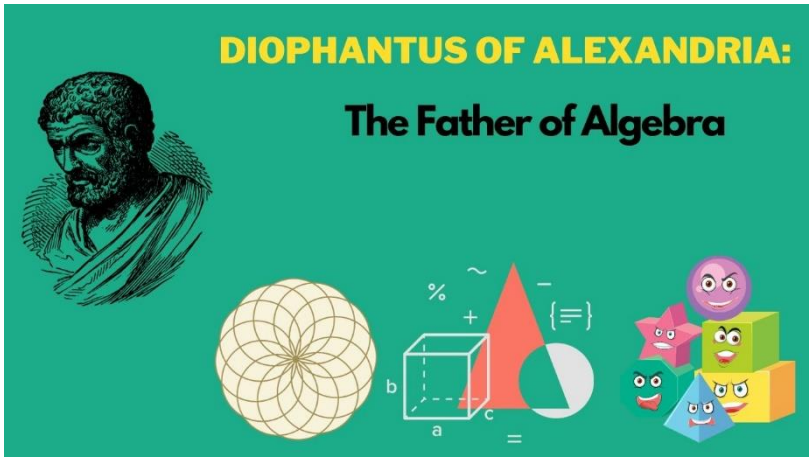
6. Look at the slides to decide what is known about Diophantus:





7. Read the text and put 3 questions to it to discuss them in the group:

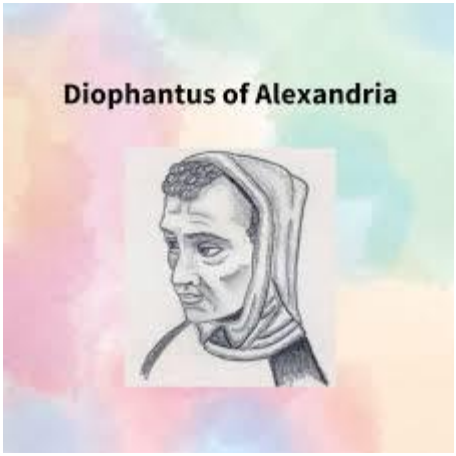
Diophantus's Contribution



Although he had limited algebraic tools at his disposal, Diophantus managed to solve a great variety of problems, and the Arithmetica inspired Arabic mathematicians such as al-Karajī (c. 980–1030) to apply his methods.

The most famous extension of Diophantus's work was by Pierre de Fermat (1601–65), the founder of modern number theory. In the margins of his copy of Arithmetica, Fermat wrote various remarks, proposing new solutions, corrections, and generalizations of Diophantus's methods as well as some conjectures such as Fermat's last theorem, which occupied mathematicians for generations to come. Indeterminate equations restricted to integral solutions have come to be known, though inappropriately, as Diophantine equations.

8. Answer the following questions for summary:



1) What was Diophantus famous for?
2) Are there accurate facts about the life of Diophantus?
3) Why is it known he that he worked in the main scientific centre of the ancient Greek world?
4) Why is it believed that he flourished during the 3rd century?
5) What came down to us under his name?
6) What his work has a historical importance?
7) What was the most famous extension of Diophantus's work?
8) What equations have come to be known as Diophantine equations?

Glossary II

A

- the ability to count – уміння рахувати
- algebra – алгебра
- algebraic number theory – алгебраїчна теорія чисел
- amicable pair of integers – дружня пара цілих чисел
- analytic number theory – аналітична теорія чисел
- the ancient Greek world – давньогрецький світ
- archaeological artifacts – археологічні артефакти
- arithmetic epigram – арифметична епіграма

B

- branch of mathematics – розділ (галузь) математики

C

- certain numbers – певні числа

D

- decomposition into prime factors – розкладання на прості множники
- determining primes – визначення простих чисел
- digital communications – цифрові комунікації
- digital computers – цифрові комп'ютери
- direct applications – безпосереднє застосування
- dot - крапка

E

- elementary number theory – елементарна теорія чисел
- to employ algebra – використовувати алгебру
- the Euclidean algorithm – алгоритм Евкліда

F

- factoring large numbers – розкладання великих чисел на множники
- finite collection of primes – скінченна колекція простих чисел
- the fundamental theorem of arithmetic – фундаментальна теорема арифметики

G

- geometric number theory – геометрична теорія чисел
- the greatest common divisor of two whole numbers – найбільший спільний дільник двох цілих чисел

M

- main scientific centre – головний науковий центр
- mathematician – математик
- modern notation – сучасна нотація
- multiplicity – кратність
- multitude – безліч

N

- number mysticism – числова містика
- number theory – теорія чисел

P

- perfect number – ідеальне число
- plural – множина
- polygonal numbers – багатокутні числа
- positive integers – додатні цілі числа
- probabilistic number theory – імовірнісна теорія чисел
- professional mathematician – професійний математик
- proofs of the theorems – докази теорем
- proper divisor – власний дільник
- property - власність

Q

- quasi-rational properties – квазіраціональні властивості

R

- regular polygon – правильний багатокутник
- to retrace some landmarks – повернути деякі орієнтири

S

- sign – знак
- solving numerical problems – розв’язування числових задач
- the sum of the proper divisors – сума власних дільників

T

- tablets – планшети
- testing conjectures – перевірка припущень
- theoretical mathematics – теоретична математика
- treatise – трактат

U

- the unique factorization theorem – основна теорема арифметики

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Contents

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