



# Fast prediction of electromigration lifetime with modified mean-time-to-failure equation

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## ABSTRACT

Black's mean-time-to-failure (MTTF) equation has been applied to predict electromigration lifetime in electronics for decades. It is an empirical equation, and at least three sets of data tested under two temperatures and two current densities are needed to determine the parameter  $n$ , activation energy  $E$ , and pre-factor  $A$  in the equation. Based on Onsager's entropy production theory, we derived a modified MTTF equation, in which  $n = 2$  becomes definite. The activation energy  $E$  is intrinsic for materials; for SnAg solder joints, we can take it as 1 eV. Therefore, we only need one set of data (one temperature and one current density) to determine the perfector  $A$ , for predicting the electromigration lifetime of the test samples. Our modified MTTF equation provides a fast and cost-saving method for accurate prediction of the electromigration lifetime for electronic products.

## 1. Introduction

The downscaling of silicon chip technology is approaching its physical limits, so how to advance the current computational trend becomes challenging. The semiconductor industry has turned to packaging technologies that stack multiple chips vertically as the alternative approach for post-Moore-era electronics [1,2]. The technology of stacking multiple chips vertically is known as 3D IC packaging by using micro-bumps and through-silicon vias (TSV). The signal processing speed can be increased, and the power consumption can be decreased.

As the packaging structure becomes complicated, electromigration tests for each component in the structure, including the BGAs, C4 solder joints, Cu interconnects, TSVs, micro-bumps, as well as the interface between them, need to be carried out. Compared with electromigration tests on traditional flip-chip packaging, the test time and cost of reliability will double or triple. Therefore, the semiconductor industry must simplify the tests, shorten the test time and save the cost. In this paper, we introduce a new MTTF equation based on Onsager's entropy production, with which we can simplify the tests and achieve a fast electromigration lifetime prediction.

## 2. Theory

Onsager's entropy production rate in irreversible processes is given as [3,4],

$$\frac{TdS}{Vdt} = JX \quad (1)$$

where  $T$  is temperature,  $dS/dt$  is the rate of entropy production,  $V$  is the test sample volume, and  $J$  and  $X$  are the conjugated flux and driving force, respectively.

In electromigration, we take MTTF as the time to accumulate the threshold entropy,  $S_{\text{threshold}}$ . Because the entropy produced by electron flow (Joule heating) is waste heat, the accumulation of entropy proceeds via the electromigration of atoms. Based on Eq. (1), the total entropy production until failure is.

$$t_{\text{failure}}^{\text{failure}} = MTTF = \frac{TS_{\text{threshold}}}{VJ_e X_e} \quad (2)$$

In electromigration, we take the atomic flux  $J_e$ , driving force  $X_e$ , and diffusivity  $D$ , as below,

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$$J_e = C \frac{D}{kT} Z^* \rho j \quad (3)$$

$$X_e = Z^* \rho j \quad (4)$$

$$D = D_0 \exp\left(-\frac{E}{kT}\right) \quad (5)$$

and we will have,

$$\begin{aligned} f_{failure} = MTF &= \frac{TS_{threshold}}{VJ_e X_e} = \frac{TS_{threshold}}{V} \frac{1}{C} \frac{kT}{D} \frac{1}{(Z^* \rho j)^2} j^{-2} \\ &= \frac{S_{threshold}}{VC} \frac{kT^2}{(Z^* \rho j)^2 D_0} \exp\left(\frac{E}{kT}\right) j^{-2} = A \left(\frac{T}{j}\right)^2 \exp\left(\frac{E}{kT}\right) \end{aligned} \quad (6)$$

### 3. Results and discussion

We recall that the traditional Black's equation for MTF in electromigration is expressed as [5]:

$$MTTF = A \left(\frac{1}{j}\right)^n \exp\left(\frac{E}{kT}\right) \quad (7)$$

With our derivation, we prove that the parameter "n" in Black's equation should be 2.

The activation energy of atomic diffusion "E" in electromigration is intrinsic for a known material. Here we list the activation energies for the common interconnect materials. The activation energy of eutectic SnAg or SnAgCu solder, eutectic SnPb solder, Cu interconnects, and Al interconnects is 1.0 [6,7], 0.8 [6,7], 0.9 [8], and 0.4 ~ 0.5 [9] eV/atom, respectively. Therefore, the parameter n and E are no longer variables in Eq. (6). The unknown parameter is the pre-factor "A", thus, we only need to carry out 1T1j (one temperature and one current) measurement to determine the MTF. Typically, semiconductor industry needs to do 3T3j. So, we have greatly reduced the time and effort in the reliability study. About the choice of 1T1j, they should be close to the user condition with a temperature of 100 °C and a current density of  $2 \times 10^3$  A/cm<sup>2</sup>, for solder joint technology. However, if the chosen testing condition is too close to the user condition, there is no acceleration effect, which is not suitable either.

Below, we attempt to verify our model of simple electromigration lifetime prediction with a direct comparison to the traditional method of using three sets of tests, including two different temperatures and two different current densities (2T2j). The Weibull distribution of the data is shown in Fig. 1 [5]. The two temperatures were 125 °C and 150 °C, and the two current densities were  $1 \times 10^4$  A/cm<sup>2</sup>, and  $5 \times 10^3$  A/cm<sup>2</sup>, respectively. The MTF (at 50 % failure) values were read from the Weibull distribution function and listed in Table 1. Using Eq. (7), the activation energy  $E = 1.15$  eV/atom, and  $n = 2.08$ . If we assume the user condition has a temperature of 100 °C, and applies a current density of  $2 \times 10^3$  A/cm<sup>2</sup>, the calculated MTF would be about 13.2 years.

Now, we use the same set of data and calculate the pre-factor "A" with the modified equation in Eq. (6). By plugging in  $n = 2$ , and  $E = 1.0$  eV, we obtain the calculated A listed in Table.1. The MTF at user condition is calculated to be 4.3 years, 5.3 years, and 38 years. If we use average A to calculate MTF, we obtain 15.9 years. Compared to MTF = 13.2 years in the traditional calculation, they are in the same range. However, the three MTF values in Table 1 have a very big difference. The first two sets of data are almost ten times smaller than the third set of data. Thus, the average is meaningless.

Next, we also used the data for eutectic sn-Pb from another publication to do the calculation [10]. The diameter of the Cu pad is around 100 μm, and the current density is  $2 \times 10^3$  A/cm<sup>2</sup>. The collected data is shown in Fig. 2, and the data obtained from the figure is listed in Table 2. We obtain  $E_a = 1.01$  eV,  $n = 1.94$ , and the MTF at user conditions of  $1 \times 10^3$  A/cm<sup>2</sup> equals to 11 years. By comparison, the calculated A with our modified MTF equation with  $n = 2$  and  $E_a = 1.0$  eV, are also listed

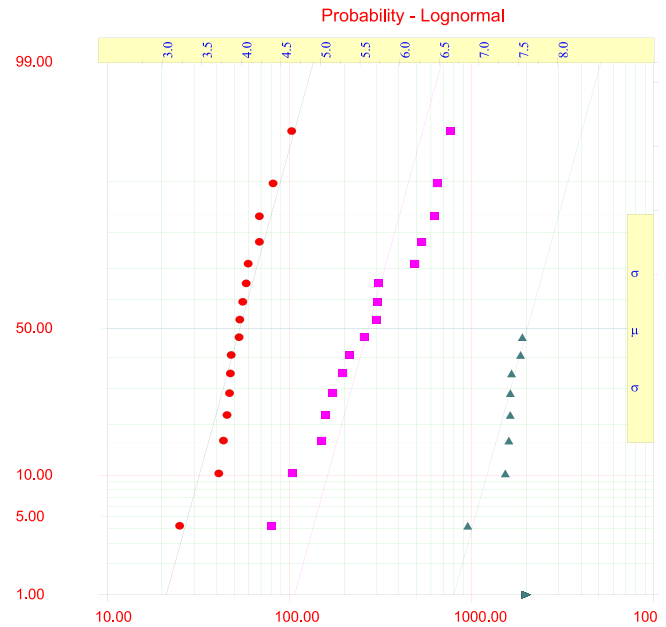


Fig. 1. Weibull distribution of three tests (2T2j) (adapted from ref. [9]).

Table 1

The electromigration tests results from Fig. 1.

Tests	1	2	3
T (°C)	150	125	150
Current density (10 <sup>4</sup> A/cm <sup>2</sup> )	1	1	0.5
MTTF (h)	48.9	288.5	1700.6
User condition MTF (years) prediction by Black's equation	13.2		
A (10 <sup>-8</sup> h·A <sup>2</sup> /K <sup>2</sup> ·cm <sup>4</sup> )	3.29	3.92	28.6
User condition MTF (years) prediction by new equation	4.3	5.3	38

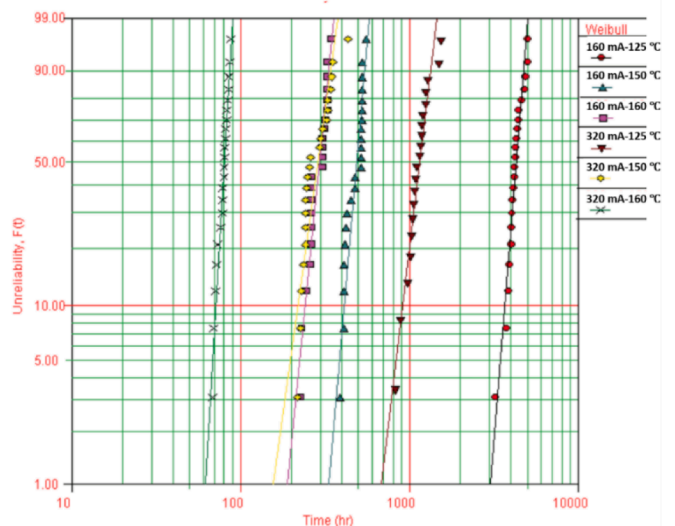


Fig. 2. Weibull plot for eutectic SnPb solder bumps (adapted from ref. [10]).

in Table 2.

In examining Table 2, we discuss here which set of data is the most reliable. The difference between the sets of data on "A" could be due to Joule heating. In our modified MTF equation, we assumed that under

**Table 2**  
The electromigration tests results from Fig. 2.

Tests	1	2	3	4	5	6
T (°C)	160	150	160	150	125	125
Current density ( $10^3$ A/cm <sup>2</sup> )	4	4	2	2	4	2
MTTF (h)	80	300	300	500	1100	4400
User condition MTTF (years)	11					
prediction by Black's equation						
A ( $10^{-8}$ h·A <sup>2</sup> /K <sup>2</sup> ·cm <sup>4</sup> )	1.55	3.21	1.45	1.35	2.39	2.39
User condition MTTF (years)	8.2	17	7.6	7.2	13	13
prediction by new equation						

steady-state Joule heating dissipates away. However, in reality, Joule heating can increase temperature. Although in electromigration tests, temperature will be calibrated and Joule heating effect is included, yet it still will be hard to know the real temperature in the test. Thus, a more accurate prediction by the modified MTTF equation will be in low current density and low temperature range, i.e. the third set of data in the first case, and the sixth set of data in the second case. They are marked bold as the last column in the Tables. The reason of choosing the low temperature and low current density is because of less Joule heating accumulation and dissipation.

In user conditions, the current density is much lower. However, in electromigration acceleration tests, we use a much higher (5 ~ 10 times) current density than the user condition. Joule heating from high current density may change the failure mode and give us unreliable data. With our modified equation, we believe that one low current density (<5000 A/cm<sup>2</sup>) and one low temperature (~125 °C) electromigration test will be enough to evaluate the electromigration lifetime of the products. Under this test condition, the electromigration failure mechanism is the closest to the user conditions, and our modified MTTF equation will save test time and cost, which will be important in advanced packaging reliability tests.

#### 4. Conclusion

Based on Onsager's entropy production theory, we derived a modified MTTF equation, in which  $n = 2$  becomes definite. With the modified MTTF equation for electromigration, only one set of data will be needed to predict the electromigration lifetime of electronic products, which can effectively save the cost and time for reliability tests.

#### CRediT authorship contribution statement

**Yingxia Liu:** Conceptualization, Methodology, Investigation,

Writing – original draft. **Andriy Gusak:** Methodology, Investigation. **Siyi Jing:** Data curation. **K.N. Tu:** Conceptualization, Methodology, Writing – review & editing.

#### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

#### Data availability

Data will be made available on request.

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