1.2 Competence Approach to Mathematics Teaching and Solving Physical Problems Graphically*

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A competence approach was put as the basis for building the contents and organization of the teaching mathematics process in the basic school. According to this approach the final result of teaching this subject is some formed competences as a pupil's abilities to apply the knowledge in studying and real life situations etc.

The present curriculum in mathematics for the 5-9 grades says[1] (2017), "teaching mathematics should make some contribution to the formation of the key competences". Namely, among the key competences they separate the basic competences in the natural sciences and technologies, which imply formation of the pupils' skill to recognize the problems arising in the environment which can be solved by the mathematical means; to build and research the mathematical models of the natural phenomena and processes etc.

We understand the subject mathematical competence as the pupils' ability to act on the basis of the received knowledge not only in exceptionally mathematical situations but also in the real ones. Therefore, supporting N. Tarasenkova[2] (2016), we consider two manifestation levels of the subject mathematical competence. Thus there are also two levels of its formation – factual level (an ability to act exceptionally in mathematical situations) and praxeological one (an ability to act in practical situations, including transferring knowledge to the other subject branches). Meanwhile, the praxeological level of the mathematics competence received by the pupils

[1] *Mathematics. 5-9 classes. Educational program for general educational establishments*. (2017). Retrieved from: https://mon.gov.ua/storage/app/media/zagalna%20serednya/programy-5-9-klas/onovlennya-12-2017/5-programa-z-matematiki.doc. [In Ukr.]

[2] Tarasenkova, N. (2016). *Competence approach in teaching mathematics: theoretical aspect. Mathematics in native school. № 11* (179). 26-30. [In Ukr.]

on some mathematical contents not very seldom transforms into the key mathematical competence, which is necessary for studying other school subjects.

The present curriculum in physics for the 7-9th grades[3] (2017) says, that "the process of the teaching physics in the basic school is aimed at the development of a pupil's personality, formation of his/her scientific outlook and the corresponding way of thinking, formation of the subject, scientific-natural (as a branch competence) and the key competences". Thus, among the key competences they defined the mathematical competence which provides for the pupils' skills: to apply mathematical methods for description, research of the physical phenomena and processes, solving physical problems, working-out and evaluation of the experiment results; to understand and use the mathematical methods for the analysis and description of the real phenomena and processes physical models; realization of the importance of the mathematical apparatus for the description and solving physical problems and tasks. The teaching resources which should be mastered by the pupils include: tasks on making calculations, algebraic transformation, making plots and pictures, analysis and presentation for the results of the experiments and laboratory works, processing of the statistic information, information presented in the plot, table and analytical forms.

A bright and efficient means for the formation of the basic school pupils' the mentioned above competences is conducting binary lessons in mathematics and physics or, which is more realistic, usage of the physical problems corresponding to some studying theme at the lessons of mathematics. From the 7th grade pupils start studying physics as well as they start studying algebra and geometry separately. Thus in the 9th grade they have a sufficient base of knowledge in all these subjects. Therefore usage of the physical problems or the problems of the physical contents at the lessons of algebra or geometry is rather actual and efficient.

[3] *Physics. 7-9 classes. Educational program for general educational establishments*. (2017). Retrieved from: https://mon.gov.ua/storage/app/media/zagalna%20serednya/programy-5-9-klas/onovlennya-12-2017/7-fizika.doc. [In Ukr.]

Many Ukrainian scientists, who deal with competence approach implementation into the educational process of the secondary education institutions, focus their attention mainly on the theoretical and methodological grounds of the problem. Contents, structure, formation of the future specialists' competence were considered in the works of N. Bibik[4] (2004), A. Khutorsky[5] (2003) and others. Many works were fixed on the development problems of the school mathematical education in Ukraine. Namely, I. Akulenko[6] (2013) researched the theoretical bases of the future teachers of mathematics' competence oriented methodical preparedness in the profession-oriented school, I. Lovianova[7] (2014) researched profession oriented teaching mathematics in the profession-oriented school, N. Tarasenkova, I. Bogatyriova, O. Kolomiets and Z. Serdiuk[8] (2015) received the considerable results in the development of the means for checking mathematical competence in the basic school etc. T. Zasyekina[9] (2016), O. Pinchuk[10] (2011) and M. Sadovyi[11] (2015) presented some methodical aspects of renovation the contents, methods, organizational forms and means of the pupils' active activity according to the competence approach in the process of teaching physics.

[4] Bibik, N. (2004). Competency Approach: Reflexive Analysis. *Competency Approach in Modern Education: World Experience and Ukrainian Perspectives: Library on Educational Policy.* Kiev. 45–50. [In Ukr.]

[5] Khutorskoy, A. (2003). Key competencies as a component of a personally oriented education paradigm. *People's Education. № 2.* 58–64. [In Ukr.]

[6] Akulenko, I. (2013). *Competency-oriented methodical preparation of the future teacher of mathematics of the profile school (theoretical aspect). Monograph.* Cherkasy: Publisher Chabanenko Yu. [In Ukr.]

[7] Lovyanova, I. (2014). *Professional training of mathematics in profile school: theoretical aspect. Monograph.* Cherkasy: Publisher Chabanenko Yu.A. [In Ukr.]

[8] Tarasenkova, N., Bogatyreva, I., Kolomiyets, O., Serdiuk, Z. (2015). Means of checking mathematical competence in the basic school. *Science and Education a New Dimension, III (26), Issue 71.* 21-25. [in Ukr.]

[9] Zasekina, T. (2016). *Formation of key and subject competences of students in the process of teaching physics.* Chisinau: Institutul de Stiinte ale Educatiel. 228-230. [In Ukr.]

[10] Pinchuk, O. (2011). Formation of the subject competences of primary school students in the process of teaching physical means of multimedia technologies: *Candidate's thesis.* Kiev [In Ukr.]

[11] Sadovyi, M. (2015). Method of formation of experimental competencies of senior pupils by means of modern experimental kits in physics. *Pedagogical sciences: theory, history, innovative technologies. Sumy.* 268-279. [In Ukr.]

It is still an acute problem in school practice to teach pupils studying mathematics to properly apply their knowledge for solving problems in other branches, namely physics, chemistry, biology, geography etc. in spite of the wide range of the pedagogical, psychological, and methodological researches (according to the questionnaire results of the pupils, teachers and students; according to the results of the state final examination and external independent testing in mathematics; according to the results of TIMSS and PISA). A teacher should correctly from the didactic point of view combine the formation of the key mathematical competences as a result of training the basic abilities and skills and the skill to use them while solving applied problems or even competence tasks. It is necessary to do this work at the lessons of mathematics, namely at the binary lessons of mathematics and physics, mathematics and biology etc. as well as during out-of-classes activity, for example, involving the contents of the optional courses, electives and mathematical circles.

According to the curriculum[12] (2017) a functional line is one of the main notions in the course of algebra in the 7-9th grades, besides it is developing in the close connection with the identical transformations, equations and inequations. As a rule the properties of the functions are defined according to their plots, i.e. on the basis of the visual imaginations, and only some properties are substantiated analytically. During teaching functions a leading place is given to the formation of the abilities to make and analyze the plots of the functions, according to the functions to characterize the processes described by them, to the ability to understand a function as a certain mathematical model of the real process. For training the pupils' knowledge, skills and abilities one should use physical problems, where real physical processes are described.

A graphical method in physics provides for the usage of plots for the description and explanation of the real processes and conformities, and it is a powerful means

[12] *Mathematics. 5-9 classes. Educational program for general educational establishments.* (2017). Retrieved from: https://mon.gov.ua/storage/app/media/zagalna% 20serednya/programy-5-9-klas/onovlennya-12-2017/5-programa-z-matematiki.doc. [In Ukr.]

for solving physical problems. It is closely connected with studying plots of functions in the course of mathematics. Studying of one and the same material in different courses mutually enriches academic subjects and fills mathematical examples with the concrete contents. This method should be used from the first lessons of the course called "The Bases of Cinematics". Graphical image of the rectilinear motion laws and the analysis of the plots allow to teach the pupils to define the motion character and numerical meanings of the path, displacement, speed and acceleration according to the plot; to visually depict functional dependences of the cinematic values; to compare the plots of motions, according to which cinematic values can be defined; to teach the pupils to solve the problems in encounter motions of the bodies (to define time and place of the encounter, speed at the moment of the encounter etc.).

Problems using plots can be divided into two kinds:

- problems, where receiving an answer is possible as a result of making a plot;
- problems, in which receiving of an answer is possible as a result of plot analysis.

An example of the first kind of problems can be the following problem.

Problem 1

A ball is feely falling down from some height. Considering its hit on the ground absolutely resilient, make a dependence of the ball motion speed against time.

Solution

We are doing an imaginary experiment. Let's image, that a ball is at some height and its initial speed equals zero. As a result of its free falling down at the moment of touching the ground, the ball had some speed ϑ. Due to absolutely resilient hit the direction of the ball speed changes into the opposite one, and it module remains the same. This happens during a very short period of time, which can approximately be considered instantaneous. The next movement of the ball is rectilinear, uniformly retarded and its final speed equals

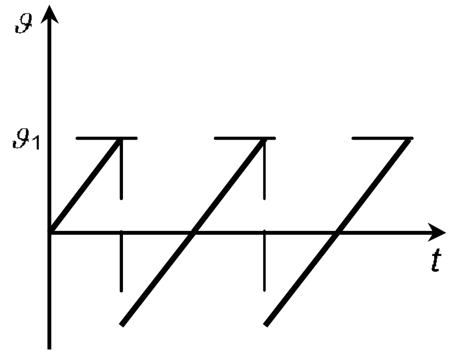

Figure 1.2.1

zero. Making a plot for dependence of the ball motion speed against time is better to draw on the blackboard by one of the students with active discussion of the audience. The plot is presented on the Figure 1.2.1.

Usage of plots contributes to the pupils' visual and deeper understanding of the physical process, it teaches them to express the functional dependence graphically, it gives an opportunity to imagine the given task, and also its solution. Since schoolchildren, even those who make plots very well, do not always see the connection of the real processes with the functional dependence.

The preference of the graphical method usage as compared with the other methods can be demonstrated by the following problem.

Problem 2

A car drove the distance between two settlements with the average speed $\vartheta_c = 54\ km/h$ during the time $t = 16\ min$. Speedup and braking lasted $t_1 = 8\ min$, and the car moved equally for the rest of the way. What speed ϑ did the car have during equable motion?

Solution

Let's depict the plot for the dependence of the car speed against time (Figure 1.2.2). The section *OA* corresponds to the speedup of the car, the section *AB* corresponds to the equable motion, and the section *BC* corresponds to its braking. The path driven by the car is numerically equal to the area of the trapezium formed by the plot of speed:

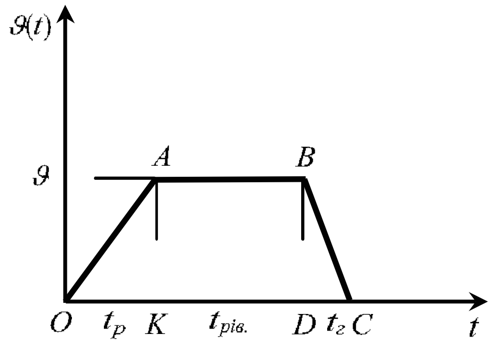

Figure 1.2.2

$$S = \frac{AB + OC}{2} AK \text{ or } S = \frac{2t_{equal} + t_1}{2} \vartheta .$$

On the other hand, from the definition of the average speed

we have: $S = \vartheta_c t$. Thus, $\vartheta = \dfrac{2\vartheta_c t}{2t - t_1}$.

In order to prepare pupils for solving the data of the physical problems it is desirable to revise with them the following:

1 Linear function, its properties and plot;
2 Areas of the main geometric figures (triangle, trapezium, parallelogram, rectangle, rhombus etc.) depending on what exactly geometric figures will be used in the offered tasks.

To prepare the pupils to solving the mentioned physical tasks it is recommended to offer them the following questions for actualization the basic knowledge:

- What function is called a linear one?
- What is the plot of the linear function?
- How many meanings of the linear function is it necessary to know for making its plot?
- How is it possible to make a plot of the linear function?
- With what meanings of k the linear function $y = kx + b$ is increasing (decreasing; stable)?
- How does number b characterizes the linear function?
- How is it possible to find zeroes of the function according to the plot?

- How can we find points of intersection the plot of the function with the coordinate axes?
- How can one define the intervals of the function increasing (decreasing) according to the plot?
- According to what formulae do they calculate an area of a random triangle?
- According to what formula do they calculate an area of an equilateral (rectangular) triangle?
- How can one prove equation of two uneven figures areas?
- According to what formula do they calculate an area of a trapezium?
- How is it possible to find an area of a trapezium, knowing its altitude and middle line?

Except traditional tasks, offered to the pupils at the lessons of mathematics for making plots of the simpler linear functions like: $y = 0{,}6x - 1{,}2$; $y = -3x + 1{,}5$, it is recommended to offer the schoolchildren to make plots of the more complicated functions[13] (2015):

$$1)\ y = |x| + x;\ 2)\ y = |x| - x;\ 3)\ y = \frac{|x|}{x};$$

$$4)\ y = \begin{cases} x-2, \text{ if } x \le -1, \\ 3x, \text{ if } -1 \le x < 2, \\ 2x+4, \text{ if } x \ge 2; \end{cases} \quad 5)\ y = \begin{cases} 5, \text{ if } x \le -2, \\ -x+3, \text{ if } -2 < x < 2, \\ 0{,}5x, \text{ if } x \ge 2. \end{cases}$$

Later pupils can be offered example problems, for instance, from the textbook[14] (2017).

Problem 3

Dependence of the distance driven by a biker against time is presented by the formula:

$$s = \begin{cases} 20t, \text{ if } 0 \le t < 1, \\ 20, \text{ if } 1 \le t < 3, \\ 50 - 10t, \text{ if } 3 \le t \le 4. \end{cases}$$

[13] Tarasenkova, N., Bogatyreva, I., Kolomiets, O., Serdiuk, Z. (2015). *Algebra: textbook for the 7th form of general education institutions.* Kiev. [In Ukr.]

[14] Tarasenkova, N., Bogatyreva, I., Kolomiets, O., Serdiuk, Z. (2017). *Algebra: textbook for the 9th form of general education institutions.* Kiev. [In Ukr.]

Make a plot of the given function and define its properties according to the plot.

Problem 4

The temperature of the piece of ice is $-3^{\circ}C$, it was heated. In 5 minutes ice melted. 10 minutes later the temperature of the melted water began changing from $0^{\circ}C$ to $3^{\circ}C$. Make a plot of the dependence of temperature against time. Define the properties of the function.

The problems of the second type contribute to the development of the pupils' basic mental actions, namely analysis and synthesis, generalization and systematization, comparison etc. and formation of their certain subject competences, since analysis and making plots result in problem solution in a general form, which enables to use this approach for solving other problems of this series. The given problems can be offered to the pupils at the binary lessons in physics and mathematics, for example, in the 9^{th} grade, when the pupils have already mastered all the knowledge, skills and abilities, which are necessary for solving the following problems. Of course, binary lessons should not be conducted very often, they require a thorough and long-lasting preparation of both teachers and pupils. But sometimes such lessons are rather efficient. Since problem 5 is offered to solve in two ways, – this can be done in different groups of pupils, then the representatives of every group will explain their solution way to the rest of the pupils, and after that all the pupils will discuss together the possibilities of using one or the other way to solve the different problems.

Problem 5

At what maximum distance l_{max} can a person be, if he is running equally with the speed ϑ in the direction of the bus, moving in the same direction with acceleration a, to be able to catch the bus?

Solution

1^{st} way. We will depict the plots of coordinate dependence against the time for a bus (1) and for a human (2, 3, 4)

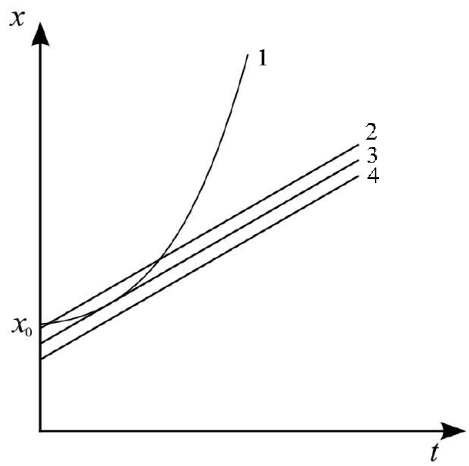

Figure 1.2.3

(Figure 1.2.3). If these plots do not intersect (1, 4), then the human will never catch the bus, two mutual points on the plot (1, 2) do not also satisfy the problem condition (the human must only catch the bus), thus the case fits, when the plot of the coordinate dependence against the time for the human is tangent to the plot of the coordinate dependence against time for the bus (1, 3). In the last case the speed of the bus equals with the human's speed at the moment of encounter.

For a deeper analysis of the given task it is necessary to make plots of the speed dependence against time for the human and for the bus (Figure 1.2.4, Figure 1.2.5).

In the Figure 1.2.4 we analyze dependence $l_{\max}$ against the human's speed change ($\vartheta_{л2} \rangle \vartheta_{л1}$). We choose the points B and B_1, which correspond to the equation of the human's speed and the speed of the bus. Since $l_{\max}$ equals the area of the triangles OAB and OA_1B_1 , we make a conclusion, that the bigger the human's speed is the bigger is $l_{\max}$.

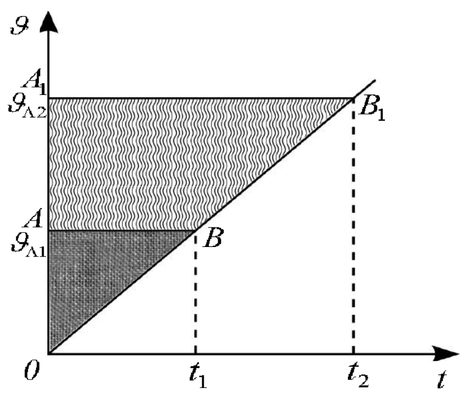

Figure 1.2.4

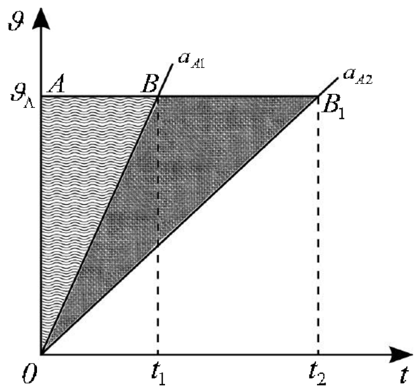

Figure 1.2.5

In the Figure 1.2.5 we consider the situation for different meanings of the bus acceleration ($a_{A1} > a_{A2}$). Comparing the areas of the triangles OAB and OAB_1 we come to the conclusion, that the bigger the acceleration of the bus is, the less is the distance l_{max}. In the Figure 1.2.4 we see, that

$l_{\max} = S_{AOB} = \frac{1}{2}\vartheta_л t$. On the other hand, acceleration of the bus

is $a_A = \frac{\vartheta_л}{t}$. As a result we get $l_{\max} = \frac{\vartheta^2}{2a}$. Thus, the given

problem can be solved by making and analyzing the plots of motion.

While solving a problem the main attention should be paid not only to the solution of the concrete problem, but also to the general approach to solving. An ability to analyze the problem and see its solution in general helps the pupils to consciously find the necessary values, instead of random search for the correct solution. All this requires a pupil's ability to analyze, to generalize, to make an imaginary experiment, to imagine physical processes, which, in their turn, contributes to forming these or those subject competences.

With the purpose of developing flexibility of thinking pupils can also be offered to solve the same problem in another way, in an algebraic one.

Solution

2nd way. We will write down the condition of the human and

bus encounter: $x_1 = x_2$ or $\vartheta t = x_0 + \frac{at^2}{2} \Rightarrow at^2 - 2\vartheta t + 2x_0 = 0$.

The condition of the single solution is that a discriminant equals zero. From this condition we find the searched

distance: $D = 4(\vartheta^2 - 2ax_0) = 0 \Rightarrow x_0 = \frac{\vartheta^2}{2a}$.

If the discriminant equals zero, x_0 has the maximum meaning, and the time when the human will catch the bus equals $t = \frac{\vartheta}{a}$.

The detailed analysis of different ways to solve the problems will allow the pupils to master one or the other approach and further use it in solving essentially new problems. For training the offered approach to solving problems which provides for the analysis of the plots the schoolchildren are given the following problem (we recommend to give it as the home-task).

Problem 6

Points A and B are located at the distance $s = 4$ km from one another. From the point A in the direction of the point B started a car, which moved all the time with equal speed. Simultaneously, another car started in its direction from the point B with the initial speed $\upsilon_0 = 32$ m/s. It moved with the constant acceleration $a = 0{,}2$ m/s^2, directed all the time in the same way as the speed of the first car. It is known that the cars twice overran each other. Within what limits is the speed of the first car?

Solution

The plot of the second car motion is a parabola (Figure 1.2.6). It is evident that the speed of the first car cannot be very big, as otherwise overtaking would happen only once (point B, while the point A corresponds to the encounter of the cars). The speed cannot also be very small (OC), because in this case the cars cannot happen to be side by side at all. It means that the equation expressing equality of the cars coordinates: $\upsilon_1 t = s - \upsilon_0 t + \frac{at^2}{2}$, should have two true solutions, besides, both of them correspond to the later moments of time, than the moment of stopping (instantaneous) of the second car: $0 = -\upsilon_0 + at$.

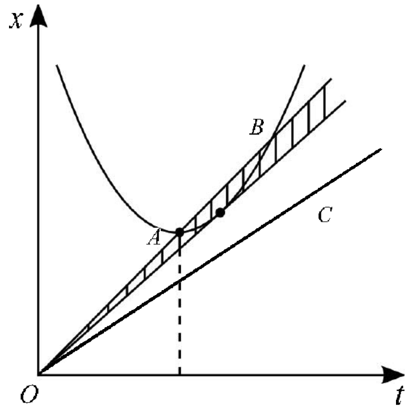

Figure 1.2.6

$$\frac{at^2}{2}-(\upsilon_1+\upsilon_0)t+s=0; \quad t^2-\frac{2(\upsilon_1+\upsilon_0)}{a}t+\frac{2s}{a}=0;$$

$$D=\left[\frac{2(\upsilon_1+\upsilon_0)}{a}\right]^2-\frac{8s}{a}>0; \quad \frac{2(\upsilon_1+\upsilon_0)}{a}>\sqrt{\frac{8s}{a}};$$

$$2(\upsilon_1+\upsilon_0)>2\sqrt{2sa}; \quad \upsilon_1>\sqrt{2sa}-\upsilon_0.$$

Taking into consideration that this speed should not be big to make a overtaking twice, consider the stopping moment of the second car $t=\frac{\upsilon_0}{a}$. Substituting this time in the equation of the motion, we have: $\upsilon_1\frac{\upsilon_0}{a}=s-\upsilon_0\frac{\upsilon_0}{a}+\frac{a}{2}\frac{\upsilon_0^2}{a};$

$$\upsilon_1=\frac{a}{\upsilon_0}s-\upsilon_0+\frac{a}{2}\frac{\upsilon_0}{a}; \quad \upsilon_1=\frac{as}{\upsilon_0}-\frac{\upsilon_0}{2}.$$

Thus, $\sqrt{2sa}-\upsilon_0<\upsilon_1<\frac{as}{\upsilon_0}-\frac{\upsilon_0}{2}$.

In general, an important basis for the pupils' formation of the subject competences (mathematical ones at the lessons of physics or physical ones at the lessons of mathematics) is making didactically balanced system of the integrated competence tasks (in mathematics and physics) for the pupils of the basic and later senior profession-oriented school. That will take into account the main tasks for studying mathematics and physics at school and will contribute to the multi-sided development of pupils.

We consider that application of such instrument as a binary lesson in related academic disciplines (physics and mathematics, mathematics and biology, mathematics and chemistry, mathematics and informatics, mathematics and geography) is rather efficient for the formation of the subject competences, mentioned in the present curricula in the mentioned academic disciplines. Making such lessons can be the subject matter of our further scientific research.