Elementary model of severe plastic deformation by KoBo process

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Self-consistent model of generation, interaction, and annihilation of point defects in the gradient of oscillating stresses is presented. This model describes the recently suggested method of severe plastic deformation by combination of pressure and oscillating rotations of the die along the billet axis (KoBo process). Model provides the existence of distinct zone of reduced viscosity with sharply increased concentration of point defects. This zone provides the high extrusion velocity. Presented model confirms that the Severe Plastic Deformation (SPD) in KoBo may be treated as non-equilibrium phase transition of abrupt drop of viscosity in rather well defined spatial zone. In this very zone, an intensive lateral rotational movement proceeds together with generation of point defects which in self-organized manner make rotation possible by the decrease of viscosity. The special properties of material under KoBo version of SPD can be described without using the concepts of nonequilibrium grain boundaries, ballistic jumps and amorphization. The model can be extended to include different SPD processes.
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I. INTRODUCTION

Last two decades were the time of fast progress of various methods of Severe Plastic Deformation (SPD).[1,2] Most popular SPD methods are equal-channel angular pressing (ECAP) and high pressure torsion (HPT). Industrial use of materials produced by SPD is limited by the low production rate, small amount of the material and inhomogeneity of structure.[3–5] Some time ago Korbel and Bochniak suggested one more way of SPD, called KoBo.[6–9] Scheme of device is shown at Fig. 1.

KoBo method resembles HPT (pressure is implied simultaneously with torsion), with very important difference: torsion is oscillating with frequency several Hertz and amplitude about 5–7°. Such a complex method of plastic straining causes highly heterogeneous flow of metals in strongly shortened zone of deformation and as a result of radial flow in the direct vicinity of the die. An associated drastic decrease in the extrusion force, which depends on frequency and amplitude of the die rotations, deforms metals with very large strains at low temperature, which make the method unique.

During the KoBo extrusion process, the metal billet undergoes reversible plastic twisting just before entering the cross section reducing die. The reversible metal twist does not affect directly the geometry of the billet, however, it transforms the typical for extrusion axio-radial flow into the layer like radial one. The measurements of the parameters of the KoBo process were performed using a press with nominal load of 1000 kN during extrusion of aluminum and hardly deformable 7075 alloy[9] (Fig. 1). Billets 65 mm long with a diameter of 40 mm were extruded to wires with diameters of 4, 6, 8, and 12 mm (the extrusion ratio: 100, 44, 25, and 11), which correspond to true strains of 4.6, 3.8, 3.2 and 2.4, respectively. The process was conducted at temperature range 293–573 K and in the case of 7075 alloy at 673 K, at a constant extrusion rate in the range of 0.07–5.0 mm/s. The cyclically rotating part of the die had an outer diameter of 35 mm and oscillated at an angle of $\pm 8^\circ$ with a constant frequency within the 2–8 Hz range. During the process, the extrusion force, torsion force acting on the die arm of the crank system and the displacement of the punch as a function of time were recorded.[9] Increased torsion frequency resulted in the decreased power consumption and the increased extrusion rate. For the press load equal 750 kN, at the die oscillation frequency of 2 Hz, the true extrusion rate stabilizes at the level of 0.07 mm/s, while the torsion force equals 6 kN×m. The increase of frequency to 5 Hz leads to a decrease of torsion force to a level of 2 kN×m and an increase of true extrusion rate to 5.0 mm/s. Consequently, the extrusion work, calculated per unit volume of aluminum, is reduced from 6.46 to 1.28 J/mm^3. The flow of material during KoBo process occurs under the influence of a much lower total deformation work, compared to the processes characterized by the homogeneously distributed point defects.

Whose experiments led to identification of the metal flow during the KoBo process as a viscous flow, which follows the Newton's law of laminar flow. They demonstrated that the metal within narrow layer around rotation zone becomes superplastic and is squeezed out practically as viscous liquid with velocity about 0.5 mm/s. In addition, the measurements of the kinetics of metal flow have shown a linear dependence of extrusion stress on strain rate, i.e., the Newtonian type of the flow: $\sigma = \eta \dot{\varepsilon}$. They proved a half order of magnitude decrease the proportionality factor (viscosity coefficient); from the $\sim 10^7$ Pa×s at the beginning of the process at room temperature to the 5×10^6 Pa×s at

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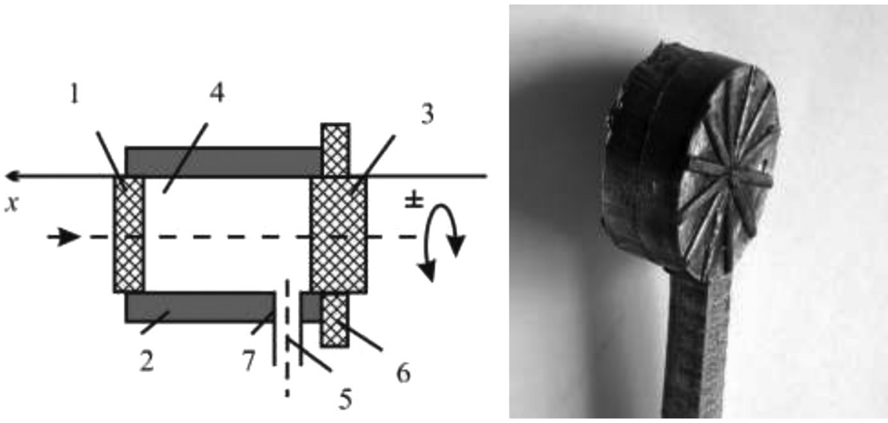

FIG. 1. Scheme of KoBo lateral extrusion and photograph of the process product: 1—punch, 2—container, 3—reversibly rotating mandrel with ragging on front surface, 4—extruded metal/billet, 5— product, 6—lock, 7—die located on container's side surface.

300 °C. Both the radial flow and linear dependence of the flow stress on the strain rate point that under such deformation conditions the polycrystalline metal behaves like a viscous fluid.[9] Hence, the stress resulting from the extrusion force, σ, and the viscosity, η, become the controlling factors for the strain rate, $\dot{\varepsilon}$, of a metal flowing through the die. The experimentally found activation enthalpy of the KoBo process coincides with the migration enthalpy of the self-interstitials (0.06 eV). Authors of this method argue that such behavior is caused by intensive generation of point defects, and first of all, interstitials which provide high plasticity.

Here, we suggest the first self-consistent phenomenological model describing generation, interactions and annihilation of point defects in the stress gradient zone of an open system. This model explains the existence of low viscosity zone.

II. MODEL FORMULATION

We consider quasi-one-dimensional (along x-axis, Fig. 1) model in which the following fields are calculated self-consistently:

- amplitude of lateral velocity $V_y(t,x)$ of oscillating rotations of material, which varies from layer to layer along x-axis within SPD zone, outside this zone $V_y = 0$;
- viscosity $\eta(t,x)$, which has much lower magnitude within the SPD zone;
- mole fraction of interstitials $N_i(t,x)$ and vacancies $N_V(t,x)$;
- hydrostatic stress component $\sigma(t,x)$, estimated as equal to one third of the Spur of stress tensor.

We derive equations for all mentioned fields. At first, consider layer of thickness dx and area S with center at x-axis. Writing down viscous forces at both boundaries of thin layer and Newton's second law for layer itself, one comes to known equation for dynamics of lateral velocity transfer:

$$\frac{\partial V_y}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial x}\left(\eta(t,x)\,\frac{\partial V_y}{\partial x}\right) \quad \text{for} \quad 0 < x < H, \tag{1}$$

where H denotes the width of SPD zone, through which the rotations are transferred.

Lateral velocity is zero at the boundary of this zone and outside it:

$$V_y(t,H) = 0; \quad V_y(t,x) = 0 \quad \text{for} \quad x > H. \tag{2}$$

Equation (1) has form of the diffusion equation (2nd Fick's law), with lateral velocity in this case being a diffusant, which is transferred (diffused) through the SPD zone. Effective diffusivity D of lateral velocity is very high (can be up to $10^3\,\mathrm{m^2/s}$), and it means that the system has to relax coming to steady-state state, when

$$\frac{\partial V_y}{\partial t} \cong 0. \tag{3}$$

It means that the expression under divergence (external derivative) in the right-hand side of Eq. (1) is constant:

$$\eta(t,x)\,\frac{\partial V_y}{\partial x} \cong \sigma_t \quad \text{for} \quad 0 < x < H. \tag{4}$$

Physical sense of σ_t is evident—it is an amplitude of tangential rotating stress within SPD zone. To provide the movement of layers under mentioned stress, this stress should reach the yield stress limit:

$$\eta(t,x)\,\frac{\partial V_y}{\partial x} \geq \sigma_{crit}. \tag{5}$$

It seems quite natural that the oscillating rotations cause the intensive reversible dislocation movements which, in turn, should lead to massive generation of the dislocation dipoles. These become the very effective source of point defects. The point defects decrease the material viscosity since the increase of point defect concentration leads to proportional increase of diffusivities. Since, the activation energy of KoBo process is very low[9] (for aluminum, it is approximately equal to 0.06 eV), it may indicate that the point defect generation plays a key role. Therefore, it seems natural to express the viscosity in terms of these defects. If viscosity would have purely diffusion nature (diffusion-controlled creep) then its dependence on defect concentration would be strictly inversely proportional. Yet, due to existence of non-diffusional viscosity (viscosity at equilibrium point defect concentration—η_o), one may assume the following type of dependence:

$$\eta(t,x) = \frac{\eta_0}{1 + \alpha N_i}, \tag{6}$$

where α is an experimentally established parameter characterizing the dependence of the viscosity on the point defects concentration.

Defects concentration evolution is determined from continuity equations:

$$\frac{\partial N_i}{\partial t} = q_i - \frac{N_i}{\tau_i} - \alpha_R D_i N_V N_i + \frac{\partial}{\partial x}\left(D_i \frac{\partial N_i}{\partial x} - N_i \Omega_i \frac{D_i}{kT}\frac{\partial \sigma}{\partial x}\right), \tag{7}$$

$$\frac{\partial N_V}{\partial t} = q_V - \frac{N_V}{\tau_V} - \alpha_R D_i N_V N_i + \frac{\partial}{\partial x}\left(D_V \frac{\partial N_V}{\partial x} + N_V \Omega_V \frac{D_V}{kT}\frac{\partial \sigma}{\partial x}\right), \tag{8}$$

q_i, q_V correspond to the rate of the defects generation[10] (probability of defect formation per lattice site per unit time), second terms in the right-hand sides of Eqs. (7) and (8) describe the point defects annihilation at dislocations (at that we assume the concentration of both types of defects to be much larger than equilibrium values), the third terms takes into account the recombination of vacancies and interstitials. The last term is just a minus divergence of diffusion flux caused by gradients of concentrations and stresses.

The α_R is the recombination rate constant that can be expressed in terms of the capture radius of an interstitial by a vacancy:[10,11] $\alpha_R \approx 1/l^2_{capture}$. At the lack of experimental data, we assumed for calculation the capture radius here as $l_{capture} \approx 1$ nm. However, more precisely, $l_{capture}$ should correspond to the separation between dislocation dipoles showing opposite sign in the layer of active dislocation slip.

The product $D_i \times \tau_i$ denotes a mean squared free path length of interstitials and it is determined by the dislocation density and by the mean distance to sinks:

$$\frac{1}{D_i \tau_i} \approx 2\pi \left(\ln \frac{L}{r}\right)^{-1} \rho^{disl}. \tag{9}$$

The generation of defects is one of the ways of energy dissipation at SPD. The total energy dissipation per unit time per unit volume is equal to $\eta(t,x)\left(\partial V_y/\partial x\right)^2$. This energy should be multiplied by efficiency r, which we estimate as 0.1. To find for which number of defects the dissipate energy will be enough, one should divide the dissipated energy by the energy of defect formation. To recalculate everything per atom, one should multiply it on atomic volume Ω. Consequently:

$$q_i = r \frac{\Omega}{E_i^{form}} \eta(t,x)\left(\frac{\partial V_y}{\partial x}\right)^2 = r \frac{\Omega}{E_i^{form}} \frac{(\sigma_{crit})^2}{\eta(t,x)} = r \frac{\Omega}{E_i^{form}} \frac{(\sigma_{crit})^2}{\eta_0}(1 + \alpha N_i), \tag{10}$$

for $0<x<$H. Obviously, outside the SPD zone the q_i is equal to zero. Equations (1)–(10) form the full self-consistent mathematical model of the process. Theoretical analysis of implicit and explicit difference methods for the evolutional differential functional equations was presented by Sapa *et al.*[12,13] In this work, we will show simple analytic approximation only.

III. ANALYTIC APPROXIMATION

1. We consider the steady-state approximation for both interstitials and vacancies. Thus,
2. We neglect the divergence terms in SPD zone.

Consequently, Eqs. (7), (8), and (10) can be rewritten in the following form:

$$q_i - \frac{N_i}{\tau_i} - \alpha_R D_i N_V N_i \approx 0, \tag{11}$$

$$q_V - \frac{N_V}{\tau_V} - \alpha_R D_i N_V N_i \cong 0, \tag{12}$$

$$q_i = q_V = q_0 (1 + \alpha N_i) = q, \tag{13}$$

where $q_0 \equiv r \frac{\Omega}{E_i^{form}} \frac{(\sigma_{crit})^2}{\eta_0}$.

Equation (13) states that the number of generated vacancies and interstitials are the same. This condition is fulfilled in radiation physics, with Frenkel pair's generation as the main mechanism of the point defects formation.[10] In KoBo process, this equality is no more guaranteed but in terms of recalled mechanism of their formation it seems logical to assume that it is satisfied. Consequently, since the probabilities of formation and annihilation of vacancy disks are approximately the same Eqs. (11)–(13) are valid only within the SPD zone. There is no defect generation outside of SPD zone and one cannot neglect the divergence terms, as was done within SPD zone. Solution of algebraic Eqs. (11)–(13) leads to the following time independent values characterizing the quasi-steady-state within SPD zone ($0 < x < H$):

$$N_V^{st} = \frac{D_i}{D_V} N_i^{st}, \tag{14}$$

$$N_i^{st} = \frac{\sqrt{\left(\frac{1}{q_0 \tau_i} - \alpha\right)^2 + 4\frac{\alpha_R D_i^2}{q_0 D_V}} - \left(\frac{1}{q_0 \tau_i} - \alpha\right)}{2\left(\frac{\alpha_R D_i^2}{q_0 D_V}\right)} = \frac{2}{\left(\sqrt{\left(\frac{1}{q_0 \tau_i} - \alpha\right)^2 + 4\frac{\alpha_R D_i^2}{q_0 D_V}} + \left(\frac{1}{q_0 \tau_i} - \alpha\right)\right)}, \tag{15}$$

$$V_y(x) = \frac{\sigma_{crit}}{\eta_0}(1 + \alpha N_i^{st})(H - x). \tag{16}$$

At that the amplitude of lateral speed is maximal at the external surface ($x = 0$).

Amplitude value of velocity is given by experiment conditions; hence Eq. (16) can be treated as an equation for the experimental determination of the SPD zone:

$$H = \frac{\eta_0 V_m}{\sigma_{crit}} \frac{1}{(1 + \alpha N_i^{st})}. \tag{17}$$

IV. NUMERICAL SOLUTION OF THE MODEL EQUATIONS UNDER STEADY-STATE APPROXIMATION

Differential Eqs. (3), (6), and (7) can be solved numerically under steady-state approximation. Without steady-state approximation the numeric solution is practically impossible, due to vast difference between the relaxation times of the lateral velocity and of the point defects (at that the vacancies appear to be the slowest component).

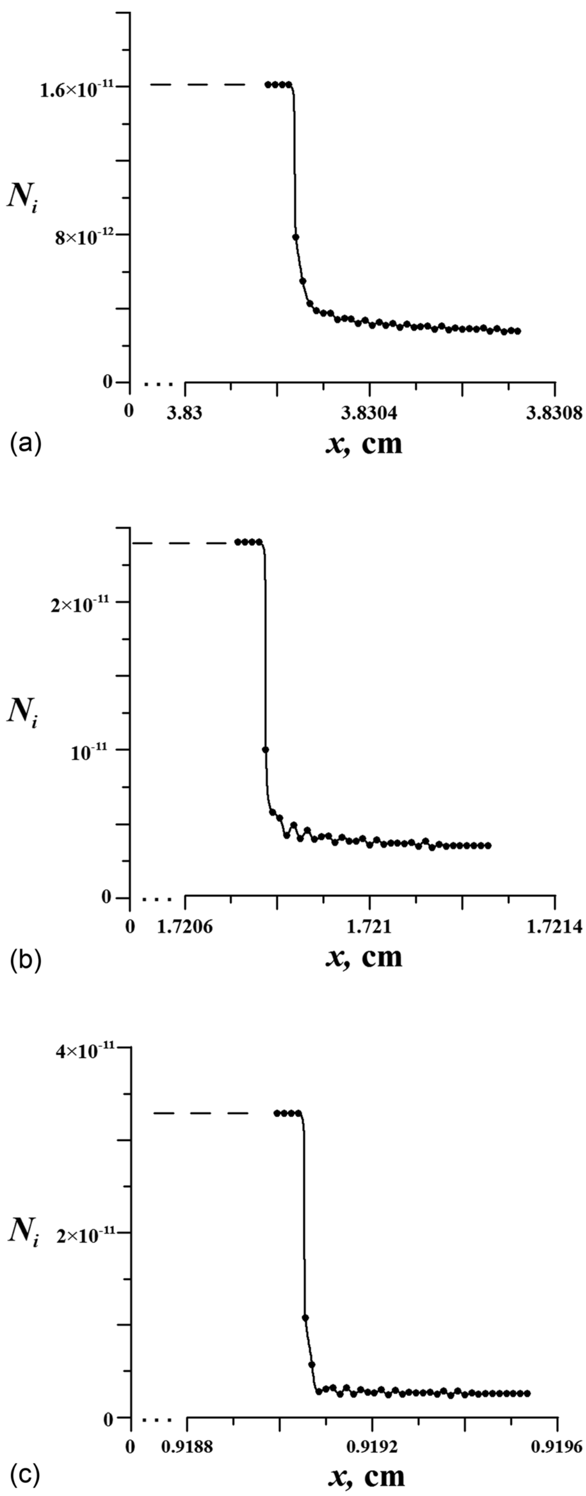

FIG. 2. Concentration profile of interstitials within the terminal part of the SPD zone: (a) $\alpha = 10^{11}$, (b) $\alpha = 2 \times 10^{11}$, (c) $\alpha = 3 \times 10^{11}$.

For numerical solution, we used the iterative scheme based on Eqs. (18)–(20), which are in fact the finite-difference form of Eqs. (3) and (6).

At first, we calculate the lateral velocity at each layer:

$$V_y[k+1] = V_y[k] - \frac{\sigma_{crit}}{\eta_0}(1 + \alpha N_i[k])\mathrm{d}x \tag{18}$$

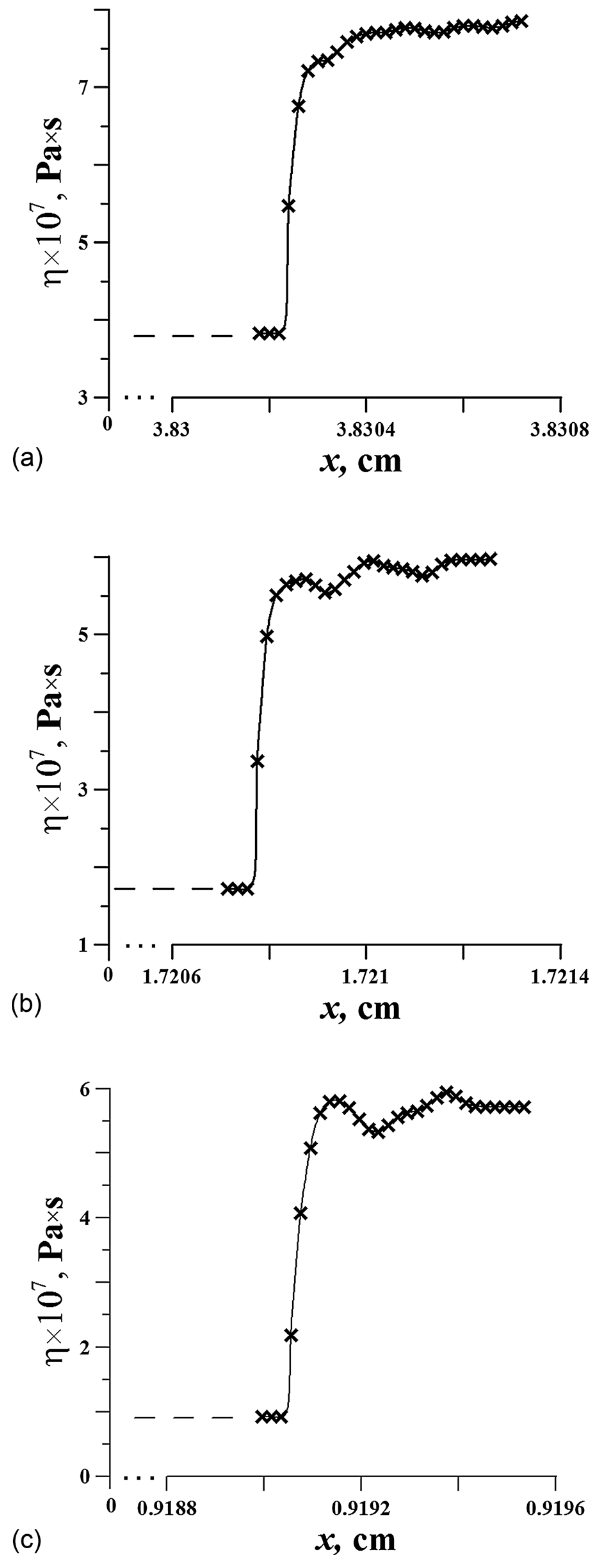

FIG. 3. Profile of viscosity within the terminal part of SPD zone: (a) $\alpha = 10^{11}$, (b) $\alpha = 2 \times 10^{11}$, (c) $\alpha = 3 \times 10^{11}$.

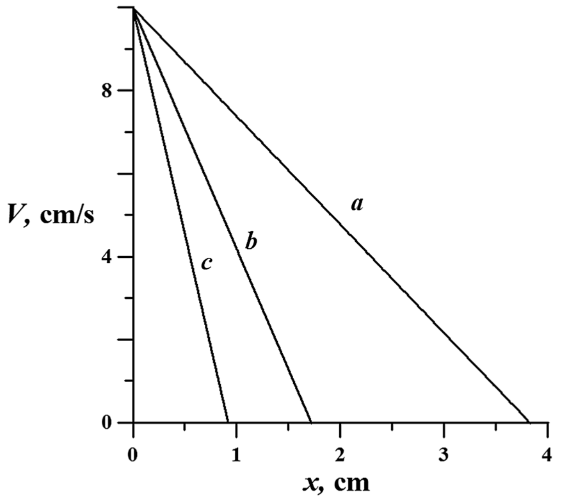

FIG. 4. Lateral velocity amplitude changing from layer to layer along x-axis within SPD zone for three values of unknown parameter: (a) $\alpha = 10^{11}$, (b) $\alpha = 2 \times 10^{11}$, (c) $\alpha = 3 \times 10^{11}$.

and fix the value $k = k_{bound}$, at which $V_y < 0$.

New concentration of interstitials is then found according to Eq. (19):

$$N_i^{new}[k] := \frac{\left(N_i[k+1] + N_i[k-1] + \frac{q_0}{D}\,dx^2\right)}{2 + \left(-\frac{q_0\,\alpha}{D} + \frac{1}{L^2} + \frac{\alpha_R D}{D_V} N_i[k]\right) dx^2}, \quad \text{if } k < k_{bound}. \tag{19}$$

Or according to Eq. (20)

$$N_i^{new}[k] := \frac{(N_i[k+1] + N_i[k-1])}{2 + \left(\frac{1}{L^2} + \frac{\alpha_R D}{D_V} N_i[k]\right) dx^2}, \quad \text{if} \quad k > k_{bound}. \tag{20}$$

Iterations are stopped if the difference between subsequent iterations becomes smaller than ε. We took $\varepsilon = 10^{-16}$.

V. RESULTS AND DISCUSSION

Let us take $\Omega = 10^{-29}\text{m}^3$, $E_i^{form} = 10^{-18}\text{J}$, $\sigma_{crit} = 10^8\text{Pa}$, $\eta_0 = 10^8\text{Pa}\times\text{s}$, $L = 10^{-6}\text{m}$, $D_i = 10^{-7}\ \text{m}^2/\text{s}$, $r = 0.1$.

Amplitude V_m of lateral velocity was evaluated by taking the radius of disc as 5 cm, period of oscillation as 0.2 s, maximal rotation angle as 6°. Then V_m is about 0.1 m/s.

The dependencies $N_i^{st}(x)$, $\eta(x)$, $V_y(x)$ calculated for various values of unknown parameter α are shown at Figs. 2–4. At the chosen parameters, the numeric results are very close to analytic ones (Table I).

TABLE I. The computed width of SPD zone during the KoBo process for different rate constants of the defects recombination.

	Analytically obtained H, m	Numerically calculated H, m
$\alpha = 10^{11}$	0.038	0.0385
$\alpha = 2 \times 10^{11}$	0.0172	0.01718
$\alpha = 3 \times 10^{11}$	0.0091	0.00913

VI. CONCLUSIONS

The above described model confirms that the Severe Plastic Deformation in KoBo process may be treated as non-equilibrium phase transition of abrupt drop of viscosity in rather well defined spatial zone. In this very zone, an intensive lateral rotational movement proceeds together with generation of point defects which in self-organized manner make rotation possible by the decrease of viscosity. Shown in Fig. 1 thickness of extrudate along x-axis of laterally extruded aluminum correlates well to the viscosity profile in Fig. 3.

Thus, the special properties of material under KoBo version of SPD can be described without using the concepts of nonequilibrium grain boundaries, ballistic jumps and amorphization. The model can be extended to include different SPD processes.

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